FIXED POINT METHODS AND OPTIMIZATION
Volume 3 (2026), No. 2, 114–129
DOI:https://doi.org/10.69829/fpmo-026-0302-ta03
Tulipa Opera Scholarum

ITERATIVE PROCEDURES AND CONTINUOUS-TIME SYSTEMS FOR VARIATIONAL INCLUSIONS VIA GRAPH CONVERGENCE OF ϕ-ACCRETIVE MAPPINGS

MOHD. FALAHAT KHAN[1], SYED SHAKAIB IRFAN[1], IQBAL AHMAD[2,*], AND MOHD. SHARIQUE[1]

[1] *Department of Mathematics, Aligarh Muslim University, Aligarh, U. P., India*
[2] *Department of Mechanical Engineering, College of Engineering, Qassim University, Saudi Arabia*

Abstract. This manuscript develops the notion of graph convergence for Φ-accretive mappings in p-uniformly smooth Banach spaces, proving its equivalence with convergence of associated resolvent and Cayley mappings. A perturbed iterative procedure is then proposed for solving variational inclusions governed by such mappings. Under suitable hypotheses, we establish both solution existence and convergence of the generated sequence, complemented by an investigation of a generalized resolvent-driven continuous-time system that elucidates trajectory behavior and equilibrium stability. Computational experiments confirm the theoretical predictions and demonstrate practical applicability.

Keywords. Resolvent Mapping, Cayley Mapping, Graph Convergence, Continuous Time System.

1. Introduction

The concept of graph convergence, originally formulated by Attouch [3] for maximal monotone mappings, establishes an essential framework for examining stability and approximation properties of set-valued maps under perturbations. Rooted in the convergence of operator graphs, this notion delivers a rigorous characterization of asymptotic behavior for sequences of mappings, particularly via their associated resolvent maps. In the classical framework, maximal monotone mappings possess resolvents that are single-valued, firmly nonexpansive, and defined on the entire space, serving crucial roles in both theoretical developments and computational implementations within nonlinear analysis and optimization; see, for instance, [6, 7, 9, 8, 11, 16, 17, 19, 13].

Subsequently, graph convergence has been generalized beyond maximal monotonicity through weaker notions of set convergence and regularization techniques. Such extensions permit treatment of broader mapping classes, including pseudomonotone, quasimonotone, and hypomonotone operators, thereby facilitating analysis of variational inequalities, equilibrium problems, and related nonlinear models. Consequently, graph convergence has emerged as an indispensable instrument for investigating operator approximation, stability, and iterative algorithm convergence in nonclassical contexts.

Along these lines, Fang and Huang [5] introduced Φ-accretive mappings in Banach spaces as a generalization of m-accretive mappings [14]. The incorporation of a nonlinear mapping Φ in the definition substantially expands the admissible mapping class, especially in Banach spaces exhibiting nonuniform geometry. Despite this generality, the corresponding generalized resolvent maps preserve essential properties such as nonexpansiveness, rendering them suitable for analyzing nonlinear evolution equations and iterative solution techniques.

*Corresponding author.
E-mail address: ssirfan.mm@amu.ac.in (S. S. Irfan), gi3635@myamu.ac.in (M. F. Khan), i.ahmad@qu.edu.sa (I. Ahmad), and gk2536@myamu.ac.in (M. Sharique)

2020 Mathematics Subject Classification: 47H04, 47H06, 47H09, 47A10.
Accepted: May 26, 2026.

On the algorithmic front, constructing efficient iterative procedures with enhanced convergence characteristics remains a central theme in nonlinear analysis. Classical single-step methods, including Picard, Krasnoselskii, and Mann iterations, constitute the foundation of fixed-point theory. When expressed through resolvent mappings, these methods can be interpreted as regularized iterations frequently exhibiting improved convergence behavior. In p-uniformly smooth Banach spaces, the geometric structure further enables refined convergence analysis for such procedures; see [15, 10, 1, 2, 20].

Motivated by these developments, this manuscript investigates graph convergence and its extensions within p-uniformly smooth Banach spaces. Building upon the theory of maximal monotone and Φ-accretive mappings, we employ generalized resolvent, averaged, and Cayley mappings to design iterative procedures with improved convergence performance. By incorporating admissible error terms into classical Picard-, Krasnoselskii-, and Mann-type iterations, we develop new algorithms exhibiting high convergence rates. Strong convergence results are obtained under appropriate hypotheses, and stability of the proposed procedures under perturbations is demonstrated.

2. Preliminaries

Let $\mathcal{E}$ denote a p-uniformly smooth Banach space equipped with norm $\|\cdot\|$. The duality pairing between $\mathcal{E}$ and its dual $\mathcal{E}^*$ is written $\langle\cdot,\cdot\rangle$, and the power set of $\mathcal{E}$ is denoted $2^{\mathcal{E}}$.

The generalized duality mapping $\mathfrak{J}_p : \mathcal{E} \to 2^{\mathcal{E}^*}$ is defined by

$$\mathfrak{J}_p(w) = \left\{ g \in \mathcal{E}^* : \langle w, g\rangle = \|w\|^p \text{ and } \|g\| = \|w\|^{p-1} \right\}, \quad \forall\, w \in \mathcal{E}.$$

When $p = 2$, the mapping $\mathfrak{J}_p$ reduces to the normalized duality mapping. Furthermore, when $\mathcal{E}$ is uniformly smooth, $\mathfrak{J}_p$ becomes single-valued.

A Banach space $\mathcal{E}$ is termed p-uniformly smooth if its modulus of smoothness $\varrho_{\mathcal{E}}(s)$ satisfies

$$\varrho_{\mathcal{E}}(s) \le \kappa\, s^p,$$

for some constants $\kappa > 0$ and $p > 1$.

Definition 2.1. A mapping $\Phi : \mathcal{E} \to \mathcal{E}$ is called

(i) accretive if

$$\langle \Phi(\alpha) - \Phi(\beta), \mathfrak{J}_p(\alpha - \beta)\rangle \ge 0 \quad \forall \alpha, \beta \in \mathcal{E};$$

(ii) strictly accretive if

$$\langle \Phi(\alpha) - \Phi(\beta), \mathfrak{J}_p(\alpha - \beta)\rangle > 0 \quad \text{whenever } \alpha \ne \beta;$$

(iii) strongly accretive if there exists $\eta > 0$ such that

$$\langle \Phi(\alpha) - \Phi(\beta), \mathfrak{J}_p(\alpha - \beta)\rangle \ge \eta\|\alpha - \beta\|^p \quad \forall \alpha, \beta \in \mathcal{E};$$

(iv) Lipschitz continuous if there exists $\sigma_\Phi > 0$ satisfying

$$\|\Phi(\alpha) - \Phi(\beta)\| \le \sigma_\Phi \|\alpha - \beta\| \quad \forall \alpha, \beta \in \mathcal{E}.$$

Definition 2.2. A set-valued mapping $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ is

(i) accretive if, for every $s \in \mathcal{T}(\alpha)$, $t \in \mathcal{T}(\beta)$, and all $\alpha, \beta \in \mathcal{E}$,

$$\langle s - t, \mathfrak{J}_p(\alpha - \beta)\rangle \ge 0;$$

(ii) m-accretive if it is accretive and $(\mathcal{I} + \mu\mathcal{T})(\mathcal{E}) = \mathcal{E}$ for every $\mu > 0$, where $\mathcal{I}$ denotes the identity mapping.

Definition 2.3. Let $\Phi : \mathcal{E} \to \mathcal{E}$ be single-valued and $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ be set-valued. We say $\mathcal{T}$ is Φ-accretive if it is accretive relative to Φ, meaning $\Phi(\alpha) + \mathcal{T}(\alpha) \ne \emptyset$ for all $\alpha \in \mathcal{E}$ and $(\Phi + \mu\mathcal{T})(\mathcal{E}) = \mathcal{E}$ for all $\mu > 0$.

The graph of a set-valued mapping $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ is given by

$$\mathrm{Graph}(\mathcal{T}) = \big\{(\alpha, \beta) \in \mathcal{E} \times \mathcal{E} : \beta \in \mathcal{T}(\alpha)\big\}.$$

Definition 2.4. [12] Let $\mathcal{T}, \mathcal{T}_n : \mathcal{E} \to 2^{\mathcal{E}}$ ($n \geq 0$) be Φ-accretive. The sequence $\{\mathcal{T}_n\}$ converges graphically to $\mathcal{T}$, written $\mathcal{T}_n \xrightarrow{\mathrm{G}} \mathcal{T}$, if for every $(\alpha, \beta) \in \mathrm{Graph}(\mathcal{T})$, there exists $(\alpha_n, \beta_n) \in \mathrm{Graph}(\mathcal{T}_n)$ with $\alpha_n \to \alpha$ and $\beta_n \to \beta$.

Definition 2.5. [4] A mapping $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ is Lipschitz continuous in the first variable if there exists $\ell > 0$ such that

$$\|\mathcal{F}(\alpha_1, \cdot) - \mathcal{F}(\alpha_2, \cdot)\| \leq \ell \|\alpha_1 - \alpha_2\| \quad \forall \alpha_1, \alpha_2 \in \mathcal{E}.$$

Lipschitz continuity in the second variable is defined similarly.

Definition 2.6. [4] Let $\Phi : \mathcal{E} \to \mathcal{E}$ and $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ be single-valued mappings on $\mathcal{E}$. Then $\mathcal{F}$ satisfies the following properties relative to Φ:

(1) $\mathcal{F}$ is γ-strongly accretive relative to Φ in its first variable if there exists $\gamma > 0$ such that

$$\big\langle \mathcal{F}(\alpha, \cdot) - \mathcal{F}(\beta, \cdot), \mathfrak{J}_p\big(\Phi(\alpha) - \Phi(\beta)\big)\big\rangle \geq \gamma \big\|\Phi(\alpha) - \Phi(\beta)\big\|^p, \quad \forall \alpha, \beta \in \mathcal{E}.$$

(2) $\mathcal{F}$ is δ-strongly accretive relative to Φ in its second variable if there exists $\delta > 0$ such that

$$\big\langle \mathcal{F}(\cdot, \alpha) - \mathcal{F}(\cdot, \beta), \mathfrak{J}_p\big(\Phi(\alpha) - \Phi(\beta)\big)\big\rangle \geq \delta \big\|\Phi(\alpha) - \Phi(\beta)\big\|^p, \quad \forall \alpha, \beta \in \mathcal{E}.$$

Definition 2.7. [18] A mapping $\mathcal{B} : \mathcal{E} \to \mathcal{E}$ is said to be θ-*averaged* for some $\theta \in (0, 1)$ if it admits the representation

$$\mathcal{B} = (1 - \theta)\mathcal{I} + \theta \mathcal{N},$$

where $\mathcal{I}$ is the identity mapping on $\mathcal{E}$ and $\mathcal{N} : \mathcal{E} \to \mathcal{E}$ is nonexpansive.

Remark 2.8. An averaged mapping is nonexpansive, meaning

$$\|\mathcal{B}(\alpha) - \mathcal{B}(\beta)\| \leq \|\alpha - \beta\|.$$

Definition 2.9. Let $\Phi : \mathcal{E} \to \mathcal{E}$ be strictly accretive and $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ be Φ-accretive. The generalized resolvent mapping is defined as

$$\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(\alpha) = [\Phi + \mu \mathcal{T}]^{-1}(\alpha), \quad \forall\, \alpha \in \mathcal{E},\ \mu > 0. \tag{2.1}$$

Definition 2.10. The generalized Cayley mapping $\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu} : \mathcal{E} \to \mathcal{E}$ is defined as

$$\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\alpha) = \left[2\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu} - \Phi\right](\alpha), \quad \forall\, \alpha \in \mathcal{E},\ \mu > 0, \tag{2.2}$$

where $\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}$ is the resolvent mapping given in (2.1), $\Phi : \mathcal{E} \to \mathcal{E}$ is single-valued, and $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ is Φ-accretive.

Proposition 2.11. *Let $\mathcal{E}$ be a uniformly smooth Banach space. Then $\mathcal{E}$ is p-uniformly smooth if and only if there exists $D_p > 0$ such that for all $\alpha, \beta \in \mathcal{E}$,*

$$\|\alpha + \beta\|^p \leq \|\alpha\|^p + p\,\langle \beta, \mathfrak{J}_p(\alpha)\rangle + D_p\,\|\beta\|^p.$$

Proposition 2.12. [5] *Let $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ be Φ-accretive, and let $\Phi : \mathcal{E} \to \mathcal{E}$ be strongly accretive with constant $\eta > 0$. Then, for any $\mu > 0$, the generalized resolvent mapping $\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}$ defined by (2.1) is Lipschitz continuous on $\mathcal{E}$ with Lipschitz constant $\frac{1}{\eta}$. Specifically,*

$$\left\|\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(\alpha) - \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(\beta)\right\| \leq \frac{1}{\eta}\left\|\alpha - \beta\right\| \quad \forall\, \alpha, \beta \in \mathcal{E}. \tag{2.3}$$

Proposition 2.13. [5] *Let $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ be Φ-accretive, and let $\Phi : \mathcal{E} \to \mathcal{E}$ be strongly accretive and Lipschitz continuous with constants $\eta > 0$ and σ_Φ, respectively. Then, the generalized Cayley mapping $\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu} : \mathcal{E} \to \mathcal{E}$ defined by (2.2) is Lipschitz continuous with constant $\psi_1 = \frac{2}{\eta} + \sigma_\Phi$. Thus,*

$$\left\|\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\alpha) - \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\beta)\right\| \leq \psi_1\|\alpha - \beta\| \quad \forall\, \alpha, \beta \in \mathcal{E}. \tag{2.4}$$

Theorem 2.14. *Let $\Phi : \mathcal{E} \to \mathcal{E}$ be strongly accretive and Lipschitz continuous with constants η and σ_Φ respectively. Suppose $\mathcal{T}_n, \mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ are Φ-accretive mappings for $n = 0, 1, 2, \ldots$. Then*

$$\mathcal{T}_n \xrightarrow{\mathrm{G}} \mathcal{T} \quad \Longleftrightarrow \quad \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(\alpha) \to \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(\alpha).$$

Proof. Assume $\mathcal{T}_n \xrightarrow{\mathrm{G}} \mathcal{T}$, i.e., for every $(u, v) \in \mathrm{Graph}(\mathcal{T})$ there exists $(u_n, v_n) \in \mathrm{Graph}(\mathcal{T}_n)$ such that $u_n \to u$ and $v_n \to v$.

Let

$$w_n = \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(u) \text{ and } w = \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(u).$$

Then,

$$\begin{aligned} w &= (\Phi + \mu\mathcal{T})^{-1}(u) \\ \frac{1}{\mu}(u - \Phi(w)) &\in \mathcal{T}(w). \end{aligned}$$

Hence

$$\left(w, \frac{1}{\mu}\Big(u - \Phi(w)\Big)\right) \in \mathrm{Graph}(\mathcal{T}).$$

By Definition 2.4 there exists $(w_n^{'}, v_n^{'}) \in \mathrm{Graph}(\mathcal{T}_n)$ such that

$$w_n^{'} \to w,\ v_n^{'} \to \frac{1}{\mu}(u - \Phi(w)) \text{ as } n \to \infty \tag{2.5}$$

and

$$v_n^{'} \in \mathcal{T}_n(w_n^{'}) \implies \Phi(w_n^{'}) + \mu v_n^{'} \in [\Phi + \mu\mathcal{T}_n]w_n^{'}$$

Thus,

$$w_n^{'} = \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}[\Phi(w_n^{'}) + \mu v_n^{'}]$$

$$\begin{aligned} \|w_n - w\| &\leq \left\|\mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(u) - \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}\Big(\Phi(w_n^{'}) + \mu v_n^{'}\Big)\right\| + \|w_n^{'} - w\| \\ &\leq \frac{1}{\eta}\left\|u - \Phi(w_n^{'}) - \mu v_n^{'}\right\| + \|w_n^{'} - w\| \\ &\leq \frac{1}{\eta}\left\|u - \Phi(w) - \mu v_n^{'}\right\| + \frac{1}{\eta}\left\|\Phi(w) - \Phi(w_n^{'})\right\| + \|w_n^{'} - w\| \\ &\leq \frac{1}{\eta}\left\|(u - \Phi(w)) - \mu v_n^{'}\right\| + \frac{\sigma_\Phi}{\eta}\left\|w_n^{'} - w\right\| + \|w_n^{'} - w\| \\ &\leq \frac{1}{\eta}\left\|(u - \Phi(w)) - \mu v_n^{'}\right\| + \left(1 + \frac{\sigma_\Phi}{\eta}\right)\|w_n^{'} - w\|. \end{aligned}$$

By (2.5), we have

$$\|w_n^{'} - w\| \to 0,\ \left\|\frac{1}{\mu}\left(u - \Phi(w)\right) - \mu v_n^{'}\right\| \to 0,$$

hence,

$$\|w_n - w\| \to 0 \text{ as } n \to \infty.$$

Conversely, suppose $\mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(u) \to \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(u)$ for all $u \in \mathcal{E}$.

For given $(u,v) \in \mathrm{Graph}(\mathcal{T})$, we have

$$\begin{aligned} \Phi(u) + \mu v &\in (\Phi + \mu\mathcal{T})(u) \\ u &= \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}[\Phi(u) + \mu v] \end{aligned}$$

Set

$$u_n = \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}[\Phi(u) + \mu v]$$

Then,

$$\frac{1}{\mu}(\Phi(u) + \mu v - \Phi(u_n)) \in \mathcal{T}_n(u_n)$$

Let $v_n = \frac{1}{\mu}(\Phi(u) + \mu v - \Phi(u_n))$, then

$$\begin{aligned} \|v_n - v\| &= \left\|\frac{1}{\mu}(\Phi(u) + \mu v - \Phi(u_n)) - v\right\| \\ &= \frac{1}{\mu}\|\Phi(u_n) - \Phi(u)\| \\ &\leq \frac{\sigma_\Phi}{\mu}\|u_n - u\|. \end{aligned}$$

Since $\mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(u_n) \to \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(u)$ for any $u \in \mathcal{E}$, we know $\|u_n - u\| \to 0$ implies $v_n \to v$ as $n \to \infty$. □

Theorem 2.15. *Let $\Phi : \mathcal{E} \to \mathcal{E}$ be strongly accretive and Lipschitz continuous with constants η and σ_Φ respectively. Suppose $\mathcal{T}_n, \mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ are Φ-accretive mappings for $n = 0, 1, 2, \ldots$. Then*

$$\mathcal{T}_n \xrightarrow{\mathrm{G}} \mathcal{T} \quad \Longleftrightarrow \quad \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(u) \to \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u).$$

Proof. From Proposition 2.13, both $\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(u)$ and $\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)$ are Lipschitz continuous. Assume $\mathcal{T}_n \xrightarrow{\mathrm{G}} \mathcal{T}$. Then for any $u \in \mathcal{E}$, let

$$w_n = \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(u), \quad w = \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)$$

Now,

$$\begin{aligned} \|w_n - w\| &= \|\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(u) - \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)\| \\ &= 2\|\mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(u) - \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(u)\| \end{aligned}$$

From Theorem 2.14 we obtain $\|w_n - w\| \to 0$ as $n \to \infty$.

Conversely, assume $\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(u) \to \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)$. For given $(u,v) \in \mathrm{Graph}(\mathcal{T})$, we have

$$\begin{aligned} \Phi(u) + \mu v &\in \Phi(u) + \mu\mathcal{T}(u) \\ u &= \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(\Phi(u) + \mu v). \end{aligned}$$

Set $u_n = \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(\Phi(u) + \mu v)$. Then we have

$$u = \frac{1}{2}\left(\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu} + \Phi\right)(\Phi(u) + \mu v) \tag{2.6}$$

$$u_n = \frac{1}{2}\left(\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu} + \Phi\right)(\Phi(u) + \mu v)\,. \tag{2.7}$$

From (2.7), we have

$$v_n = \frac{1}{\mu}(\Phi(u) - \Phi(u_n) + \mu v) \in \mathcal{T}_n(u_n).$$

Therefore,

$$\begin{aligned}\|v_n - v\| &= \frac{1}{\mu}\|\Phi(u_n) - \Phi(u)\| \\ &\leq \frac{\sigma_\Phi}{\mu}\|u_n - u\|. \end{aligned} \tag{2.8}$$

Since $\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(u) \to \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)$ for any $u \in \mathcal{E}$ and $\|u_n - u\| \to 0$, from (2.8) it follows that

$$v_n \to v \quad \text{as} \quad n \to \infty,$$

hence $\mathcal{T}_n \xrightarrow{\mathrm{G}} \mathcal{T}$. □

3. Problem Formulation and Well-Posedness

In this section, we demonstrate that the introduced graph convergence concept for Φ-accretive mappings plays a significant role in addressing variational inclusion problems within Banach spaces. Let $\mathcal{B} : \mathcal{E} \to \mathcal{E}$ be a θ-averaged mapping, $\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu} : \mathcal{E} \to \mathcal{E}$ the Cayley mapping, $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ a Φ-accretive mapping, and $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ a given mapping. We consider the problem of finding $\hat{y} \in \mathcal{E}$ such that:

$$0 \in \mathcal{F}\Big(\mathcal{B}(\hat{y}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y})\Big) + \mathcal{T}(\hat{y}). \tag{3.1}$$

Lemma 3.1. *Let $\Phi : \mathcal{E} \to \mathcal{E}$ be strongly accretive and $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ be Φ-accretive. Suppose $\mathcal{B} : \mathcal{E} \to \mathcal{E}$ is θ-averaged and $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ is a given nonlinear mapping. Then $\hat{y} \in \mathcal{E}$ solves the variational inclusion (3.1) if and only if $\hat{y}$ satisfies the fixed-point equation*

$$\hat{y} = \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(\hat{y}) - \mu\, \mathcal{F}\big(\mathcal{B}(\hat{y}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y})\big)\big), \tag{3.2}$$

where $\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}$ is the resolvent mapping and $\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}$ is the corresponding Cayley mapping.

We now establish existence and uniqueness of solutions for the fixed-point formulation of problem (3.1), confirming its well-posedness.

Theorem 3.2. *Let $(\mathcal{E}, \|\cdot\|)$ be a p-uniformly smooth Banach space and $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ be Φ-accretive. Suppose $\mathcal{B} : \mathcal{E} \to \mathcal{E}$ is θ-averaged nonexpansive, the resolvent mapping $\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}$ and Cayley mapping $\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}$ are Lipschitz continuous with constants $1/\eta$ and ψ_1 , respectively, and $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ is Lipschitz continuous in each variable with constants $\omega_1, \omega_2 > 0$ and strongly accretive in each variable with constants $\gamma, \delta > 0$. If the following condition holds*

$$\Lambda = \frac{1}{\eta}\left(\sigma^p_\Phi + D_p\mu^p(\omega_2\psi_1 + \omega_1)^p - p\mu(\gamma+\delta)\sigma^p_\Phi\right)^{\frac{1}{p}} \quad \textit{and} \quad 0 < \Lambda < 1,$$

then the variational inclusion (3.1) possesses a unique solution in $\mathcal{E}$.

Proof. Define $\mathcal{G} : \mathcal{E} \to \mathcal{E}$ by $\mathcal{G}(\hat{y}) = \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\left(\Phi(\hat{y}) - \mu\mathcal{F}(\mathcal{B}(\hat{y}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))\right)$

$$\begin{aligned}\|\mathcal{G}(\hat{y}) - \mathcal{G}(\hat{z})\| &= \left\|\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\left(\Phi(\hat{y}) - \mu\mathcal{F}(\mathcal{B}(\hat{y}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))\right) - \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\left(\Phi(\hat{z}) - \mu\mathcal{F}(\mathcal{B}(\hat{z}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\right\| \\ &\leq \frac{1}{\eta}\left\|\left(\Phi(\hat{y}) - \mu\mathcal{F}(\mathcal{B}(\hat{y}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))\right) - \left(\Phi(\hat{z}) - \mu\mathcal{F}(\mathcal{B}(\hat{z}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\right\| \\ &\leq \frac{1}{\eta}\left\|\Phi(\hat{y}) - \Phi(\hat{z}) - \mu\left(\mathcal{F}(\mathcal{B}(\hat{y}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y})) - \mathcal{F}(\mathcal{B}(\hat{z}), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\right\|.\end{aligned}$$

Applying Proposition 2.11, we obtain

$$\begin{aligned}
&\left\|\Phi(\hat{y})-\Phi(\hat{z})-\mu\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\right\|^{p}\\
\leq&\left\|\Phi(\hat{y})-\Phi(\hat{z})\right\|^{p}+D_p\mu^p\left\|\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\right\|^{p}\\
&-p\mu\left\langle\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right),\mathfrak{J}_p((\Phi(\hat{y})-\Phi(\hat{z}))\right\rangle.
\end{aligned}$$

Expanding,

$$\begin{aligned}
&\left\langle\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right),\mathfrak{J}_p((\Phi(\hat{y})-\Phi(\hat{z}))\right\rangle\\
=&\left\langle\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))+\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))-\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right),\mathfrak{J}_p((\Phi(\hat{y})-\Phi(\hat{z}))\right\rangle\\
=&\left\langle\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right),\mathfrak{J}_p((\Phi(\hat{y})-\Phi(\hat{z}))\right\rangle\\
&+\left\langle\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right),\mathfrak{J}_p((\Phi(\hat{y})-\Phi(\hat{z}))\right\rangle\\
\geq&\delta\|\Phi(\hat{y})-\Phi(\hat{z})\|^p+\gamma\|\Phi(\hat{y})-\Phi(\hat{z})\|^p\\
\geq&(\delta+\gamma)\sigma^p_\Phi\|\hat{y}-\hat{z}\|^p.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&\left\|\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right\|\\
\leq&\left\|\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))+\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))-\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right\|\\
\leq&\left\|\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right\|+\left\|\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right\|\\
\leq&\omega_2\psi_1\|\hat{y}-\hat{z}\|+\omega_1\|\hat{y}-\hat{z}\|\\
\leq&(\omega_2\psi_1+\omega_1)\|\hat{y}-\hat{z}\|^p.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\|\Phi(\hat{y})-\Phi(\hat{z})-\mu\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\|\\
\leq&\left(\sigma^p_\Phi+D_p\mu^p(\omega_2\psi_1+\omega_1)^p-p\mu((\delta+\gamma)\sigma^p_\Phi)\right)^{\frac{1}{p}}\|\hat{y}-\hat{z}\|.
\end{aligned}\tag{3.3}$$

Hence,

$$\begin{aligned}
&\left\|\Phi(\hat{y})-\Phi(\hat{z})-\mu\left(\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))-\mathcal{F}(\mathcal{B}(\hat{z}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{z}))\right)\right\|\\
\leq&\frac{1}{\eta}\left(\sigma^p_\Phi+D_p\mu^p(\omega_2\psi_1+\omega_1)^p-p\mu((\delta+\gamma)\sigma^p_\Phi)\right)^{\frac{1}{p}}\|\hat{y}-\hat{z}\|
\end{aligned}\tag{3.4}$$

$$\|\mathcal{G}(\hat{y})-\mathcal{G}(\hat{z})\| \leq \frac{1}{\eta}\left(\sigma^p_\Phi+D_p\mu^p(\omega_2\psi_1+\omega_1)^p-p\mu((\delta+\gamma)\sigma^p_\Phi)\right)^{\frac{1}{p}}\|\hat{y}-\hat{z}\| \tag{3.5}$$

$$\|\mathcal{G}(\hat{y})-\mathcal{G}(\hat{z})\| \leq \Lambda\|\hat{y}-\hat{z}\|, \tag{3.6}$$

where $\Lambda=\frac{1}{\eta}\left(\sigma^p_\Phi+D_p\mu^p(\omega_2\psi_1+\omega_1)^p-p\mu((\delta+\gamma)\sigma^p_\Phi)\right)^{\frac{1}{p}}$. By assumption $0<\Lambda<1$, the mapping $\hat{y}=\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\left(\Phi(\hat{y})-\mu\mathcal{F}(\mathcal{B}(\hat{y}),\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(\hat{y}))\right)$ has a unique fixed point $\hat{y}$ in $\mathcal{E}$. Thus $\hat{y}$ solves problem (3.1). □

4. Iterative Procedure and Convergence Analysis

The fixed-point formulation (3.2) enables construction of an iterative procedure for solving problem (3.1).

Let $\Phi,\mathcal{F},\mathcal{B},\mathcal{T},\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}$ be as in problem (3.1) with Φ strongly accretive, $\mathcal{F}$ a bifunction, and $\mathcal{B}$ a θ-averaged mapping. Then for any given $y_0\in\mathcal{E}$, we compute the sequence $\{s_n\}_{n=0}^{\infty}$ in $\mathcal{E}$ via the following iterative procedure:

Algorithm Composite Resolvent Iteration

Input: Initial point $y_0 \in \mathcal{E}$, step size $\mu > 0$

Iteration: For $n = 0, 1, 2, \ldots$, compute:

$$\begin{aligned} s_n &= \mathfrak{P}^{\mathcal{T}_n}_{\Phi,\mu}(y_n), \\ y_{n+1} &= \Phi(s_n) - \mu \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) + \varepsilon_n, \end{aligned} \tag{4.1}$$

where $\{\varepsilon_n\}_{n=0}^{\infty} \subset \mathcal{E}$ is an error term accounting for possible inexact computation satisfying:

$$\sum_{j=1}^{\infty} \|\varepsilon_j - \varepsilon_{j-1}\| \rho^{j-1} < \infty \quad \forall \rho \in (0,1) \quad \text{and} \quad \lim_{n\to\infty} \varepsilon_n = 0.$$

Theorem 4.1. *Let $\mathcal{E}$ be a p-uniformly smooth Banach space and $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ be single-valued with $\mathcal{T}_n, \mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ being Φ-accretive mappings such that $\mathcal{T}_n \xrightarrow{G} \mathcal{T}$. Suppose all hypotheses of Theorem 3.2 hold. Let $\{s_n\}$ and $\{y_n\}$ be sequences generated by Algorithm and if the following condition hold*

$$\Pi = \frac{1}{\eta} \sqrt[p]{\sigma_\Phi^p - p\mu(\gamma + \delta)\, \sigma_\Phi^p + \mu^p D_p(\omega_1 + \omega_2 \psi_1)},$$

where $0 < \Pi < 1$ then the sequence $\{s_n\}$ generated by Algorithm converges strongly to the unique solution $u \in \mathcal{E}$ of the variational inclusion problem (3.1).

Proof. Using Algorithm , we have

$$\begin{aligned} \|y_{n+1} - y_n\| &= \Big\| \big(\Phi(s_n) - \mu\, \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) + \varepsilon_n\big) \\ &\quad - \big(\Phi(s_{n-1}) - \mu\, \mathcal{F}(\mathcal{B}(s_{n-1}), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) + \varepsilon_{n-1}\big) \Big\| \\ &\le \Big\| \Phi(s_n) - \Phi(s_{n-1}) - \mu\big[\mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) \\ &\quad - \mathcal{F}(\mathcal{B}(s_{n-1}), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1}))\big] \Big\| \\ &\quad + \|\varepsilon_n - \varepsilon_{n-1}\|. \end{aligned} \tag{4.2}$$

Using Proposition 2.11, we obtain

$$\begin{aligned} &\Big\| \Phi(s_n) - \Phi(s_{n-1}) - \mu\big[\mathcal{F}(\mathcal{B}(s_n),\, \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_{n-1}),\, \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1}))\big] \Big\|^p \\ \le& \Big\| \Phi(s_n) - \Phi(s_{n-1}) \Big\|^p + D_p \mu^p \Big\| \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_{n-1}), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) \Big\|^p \\ &- p\mu \Big\langle \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_{n-1}), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})), \mathfrak{J}_p(\Phi(s_n) - \Phi(s_{n-1})) \Big\rangle. \end{aligned} \tag{4.3}$$

Using Lipschitz continuity of the accretive mapping,

$$\Big\| \Phi(s_n) - \Phi(s_{n-1}) \Big\| \le \sigma_\Phi \|s_n - s_{n-1}\|. \tag{4.4}$$

Expanding the second term of (4.3):

$$\begin{aligned} &\Big\| \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_{n-1}), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) \Big\| \\ \le& \Big\| \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) \Big\| \\ &+ \Big\| \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) - \mathcal{F}(\mathcal{B}(s_{n-1}), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) \Big\| \\ &+ \Big\| \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) - \mathcal{F}(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) \Big\|. \end{aligned}$$

Using Theorem 2.15, Lipschitz continuity of the Cayley mapping and $\mathcal{F}(\cdot,\cdot)$ in both variables, plus nonexpansiveness of the averaged mapping, we have

$$\begin{aligned}&\omega_2\Big\|\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1}))\Big\| + \omega_1\Big\|\mathcal{B}(s_n) - \mathcal{B}(s_{n-1})\Big\| + \omega_2\Big\|\mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(s_{n-1})) - \mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1}))\Big\|\\ \le&\omega_1\|s_n - s_{n-1}\| + \omega_2\psi_1\|s_n - s_{n-1}\|\\ \le&(\omega_1+\omega_2\psi_1)^p\|s_n - s_{n-1}\|. \end{aligned} \tag{4.5}$$

Expanding the third term of (4.3) and using Definition 2.6 with Lipschitz continuity of Φ,

$$\begin{aligned}&\Big\langle \mathcal{F}(\mathcal{B}(s_n),\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_{n-1}),\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})), \mathfrak{J}_p\big(\Phi(s_n)-\Phi(s_{n-1})\big)\Big\rangle\\ =&\Big\langle \mathcal{F}(\mathcal{B}(s_n),\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})) - \mathcal{F}(\mathcal{B}(s_{n-1}),\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})), \mathfrak{J}_p\big(\Phi(s_n)-\Phi(s_{n-1})\big)\Big\rangle,\\ &+\Big\langle \mathcal{F}(\mathcal{B}(s_n),\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_n),\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1})), \mathfrak{J}_p\big(\Phi(s_n)-\Phi(s_{n-1})\big)\Big\rangle\\ \le&(\gamma+\delta)\sigma^p_\Phi\|s_n-s_{n-1}\|^p. \end{aligned} \tag{4.6}$$

Combining (4.3)-(4.5):

$$\begin{aligned}&\Big\|\Phi(s_n)-\Phi(s_{n-1}) - \mu\big[\mathcal{F}(\mathcal{B}(s_n),\,\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_n)) - \mathcal{F}(\mathcal{B}(s_{n-1}),\,\mathfrak{K}^{\mathcal{T}_n}_{\Phi,\mu}(s_{n-1}))\big]\Big\|\\ \le&\sqrt[p]{\sigma^p_\Phi - p\mu(\gamma+\delta)\,\sigma^p_\Phi + \mu^p D_p(\omega_1+\omega_2\psi_1)^p}\|s_n-s_{n-1}\|. \end{aligned} \tag{4.7}$$

From (4.2):

$$\|s_{n+1}-s_n\| \quad\le\quad \Pi\|s_n-s_{n-1}\| + \frac{1}{\eta}\|\varepsilon_n-\varepsilon_{n-1}\|,$$

where $\Pi = \frac{1}{\eta}\sqrt[p]{\sigma^p_\Phi - p\mu(\gamma+\delta)\,\sigma^p_\Phi + \mu^p D_p(\omega_1+\omega_2\psi_1)^p}$. Now,

$$\begin{aligned}&\|s_{n+1}-s_n\|\\ \le&\Pi\,\|s_n-s_{n-1}\| + \frac{1}{\eta}\|\varepsilon_n-\varepsilon_{n-1}\|\\ \le&\Pi\,\left[\Pi\left(\|s_n-s_{n-1}\| + \frac{1}{\eta}\|\varepsilon_n-\varepsilon_{n-1}\right)\right] + \frac{1}{\eta}\|\varepsilon_n-\varepsilon_{n-1}\|\\ \le&\Pi^2\|s_{n-1}-s_{n-2}\| + \frac{\Pi}{\eta}\|\varepsilon_{n-1}-\varepsilon_{n-2}\| + \frac{1}{\eta}\|\varepsilon_n-\varepsilon_{n-1}\|\\ \le&\cdots\\ \le&\Pi^{n-n_0}\|s_{n_0+1}-s_{n_0}\| + \sum_{i=1}^{n-n_0}\frac{\Pi^{i-1}}{\eta}\|\varepsilon_{n-(i-1)}-\varepsilon_{n-i}\|. \end{aligned}$$

For $m \geq n > n_o$:

$$\begin{aligned}
&\|s_m - s_n\| \\
&\leq \sum_{j=n}^{m-1} \|s_{j+1} - s_j\| \\
&\leq \sum_{j=n}^{m-1} \Pi^{j-n_0} \|s_{n_0+1} - s_{n_0}\| + \sum_{j=n}^{m-1} \sum_{i=1}^{j-n_0} \frac{\Pi^{i-1}}{\eta} \left\| \varepsilon_{j-(i-1)} - \varepsilon_{j-1} \right\| \\
&\leq \sum_{j=n}^{m-1} \Pi^{j-n_0} \|s_{n_0+1} - s_{n_0}\| + \sum_{j=n}^{m-1} \left[\sum_{i=1}^{j-n_0} \frac{1}{\eta} \frac{\|\varepsilon_{j-(i-1)} - \varepsilon_{j-1}\|}{\Pi^{j-(i-1)}} \right].
\end{aligned}$$

Since $\Pi \in (0,1)$, for any $m \geq n > n_o$ we have $|s_m - s_n| \to 0$ as $n \to \infty$, so $\{s_n\}_{n=0}^{\infty}$ is Cauchy in $\mathcal{E}$. By completeness, there exists $u \in \mathcal{E}$ such that $s_n \to u$ as $n \to \infty$. Using Lipschitz continuity of the resolvent mapping:

$$\left\| \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(y_n) - \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(y) \right\| \leq \frac{1}{\eta} \|y_n - y\|. \tag{4.8}$$

Since Φ and $\mathcal{F}$ are continuous, $s_n \to u$, and $\varepsilon_n \to 0$ as $n \to \infty$, applying the algorithm shows $y_n \to y$, hence (4.8) tends to zero as $n \to \infty$:

$$\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(y_n) \to \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(y) \quad \text{as} \quad n \to \infty.$$

Therefore $u = \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(y)$ and by (3.2), u solves problem (3.1). ☐

5. Generalized Resolvent-Based Continuous-Time System

In this section, we adopt a continuous-time systems perspective to investigate solvability of problem (3.1). Using formulation (3.2), we introduce and analyze the associated generalized resolvent continuous-time system (GRCTS):

$$\frac{du}{dt} = \nu \Big\{ \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu \mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) - u \Big\}, \qquad u(t_0) = u_0 \in \mathcal{E}, \tag{5.1}$$

where $u \in \mathcal{E}$ and $\nu > 0$ is a parameter.

Definition 5.1. The GRCTS in (5.1) converges to the solution set $\Omega(\mathcal{F}, \mathcal{T}, \Phi, \mathcal{B}, \mu)$ of (3.1) if its trajectory, regardless of initial condition, satisfies

$$\lim_{t \to \infty} \operatorname{dist}(u(t), \Omega) = 0,$$

where

$$\operatorname{dist}(u(t), \Omega) = \inf_{v \in \Omega} \|u(t) - v\|.$$

If u^* is the unique point of Ω, then the trajectory converges to u^*:

$$\lim_{t \to \infty} u(t) = u^*.$$

Definition 5.2. The continuous-time system is globally exponentially stable of degree $\varkappa$ at u^* if, for any initial condition, its trajectory satisfies

$$\|u(t) - u^*\| \leq c_0 \|u(t_0) - u^*\| \exp\{-\varkappa(t - t_0)\}, \quad t \geq t_0,$$

where $\varkappa > 0$ and $c_0 > 0$ are constants independent of the initial point.

Lemma 5.3. *Let $\tilde{u}$ and $\tilde{v}$ be real-valued nonnegative continuous functions with domain $\{t : t \geq t_0\}$ and let $\phi(t) = \phi_0(|t - t_0|)$ where ϕ_0 is monotonically increasing. If for all $t \geq t_0$,*

$$\tilde{u}(t) \leq \phi(t) + \int_{t_0}^{t} \tilde{u}(\tau)\tilde{v}(\tau)\, d\tau,$$

then

$$\tilde{u}(t) \leq \phi(t) \exp\left\{\int_{t_0}^{t} \tilde{v}(\tau)\, d\tau\right\}.$$

Employing lem 5.3 together with Theorem 3.2, we establish the unique solution of GRCTS (5.1).

Theorem 5.4. *Suppose Theorem 3.2 holds. Then, for any initial point $u_0 \in \mathcal{E}$, the GRCTS (5.1) admits a unique continuous solution $u(t)$ defined on $[t_0, \infty)$ with $u(t_0) = u_0$.*

Proof. Define

$$\mathcal{G}(u) = \nu\Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)\big) - u\Big\}, \quad \forall u \in \mathcal{E}.$$

Now,

$$\begin{aligned}
\|\mathcal{G}(u) - \mathcal{G}(u^*)\| &= \nu\Big\|\Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)\big) - u\Big\} \\
&\qquad \Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u^*) - \mu\mathcal{F}(\mathcal{B}(u^*), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u^*))\big) - u^*\Big\}\Big\| \\
&\leq \nu\Big\|\Phi(u) - \Phi(u^*) - \mu\left(\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)) - \mathcal{F}(\mathcal{B}(u^*), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u^*))\right)\Big\| \\
&\qquad + \nu\|u - u^*\|.
\end{aligned} \tag{5.2}$$

By arguments analogous to (3.4):

$$\begin{aligned}
&\Big\|\Phi(u) - \Phi(u^*) - \mu\left(\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)) - \mathcal{F}(\mathcal{B}(u^*), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u^*))\right)\Big\| \\
&\leq \frac{1}{\eta}\left(\sigma^p_{\Phi} + D_p\mu^p(\omega_2\psi_1 + \omega_1)^p - p\mu((\delta+\gamma)\sigma^p_{\Phi})\right)^{\frac{1}{p}} \|u - u^*\|
\end{aligned} \tag{5.3}$$

Combining (5.2) and (5.3):

$$\|\mathcal{G}(u) - \mathcal{G}(u^*)\| \leq \nu(1 + \Lambda)\|u - u^*\|, \tag{5.4}$$

establishing that $\mathcal{G}$ is locally Lipschitz continuous on $\mathcal{E}$. Hence, for any initial point $u_0 \in \mathcal{E}$, the GRCTS (5.1) admits a unique continuous solution $u(t)$ with $u(t_0) = u_0$ on $t_0 \leq t < T$.

Let $[t_0, T)$ be the maximal interval. We show $T = \infty$. For any $u \in \mathcal{E}$:

$$\begin{aligned}
\|\mathcal{G}(u)\| &= \nu\Big\|\Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u(t)), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) - u\Big\}\Big\| \\
&\leq \nu\Big\|\Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) - u\Big\} - u^*\Big\| + \nu\|u - u^*\| \\
&= \nu\Big\|\Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u(t)) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) - u\Big\} \\
&\quad - \Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u^*) - \mu\mathcal{F}(\mathcal{B}(u^*), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u^*))\big)\Big\}\Big\| + \nu\|u - u^*\| \\
&\leq \nu\Big\|\Phi(u) - \Phi(u^*) - \mu\left(\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u)) - \mathcal{F}(\mathcal{B}(u^*), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u^*))\right)\Big\| + \nu\|u - u^*\| \\
&\leq \nu(1 + \Lambda)\|u - u^*\| \\
&\leq \nu(1 + \Lambda)\|u\| + \nu(1 + \Lambda)\|u^*\|.
\end{aligned} \tag{5.5}$$

Integrating (5.5) over $[t_0, t]$ and applying Lemma 5.3:

$$\|u(t)\| \leq \|u_0\| + \int_{t_0}^{t} \|\mathcal{G}(u(\tau))\|\, d\tau \leq \|u_0\| + \phi_1(t - t_0) + \phi_2\int_{t_0}^{t} \|u(\tau)\|\, d\tau, \tag{5.6}$$

for all $t \in [t_0, T)$, where

$$\phi_1 = \nu(1+\Lambda)\|u^*\|, \qquad \phi_2 = \nu(1+\Lambda).$$

Hence,

$$\|u(t)\| \leq (\|u_0\| + \phi_1(t-t_0)) \exp\{\phi_2(t-t_0)\}, \qquad t \in [t_0, T),$$

implying the solution is bounded on $[t_0, T)$, thus $T = \infty$. □

Theorem 5.5. *Suppose Theorem 3.2 holds. Then the GRCTS (5.1) converges globally exponentially to the unique solution $u^* \in \Omega(\mathcal{F}, \mathcal{T}, \Phi, \mathcal{B}, \mu)$.*

Proof. Theorem 5.4 establishes that GRCTS (5.1) has a unique solution. Let $u(t) = (t, t_0; u_0)$ denote a solution satisfying $u(t_0) = u_0$. Define the Lyapunov function $\mathcal{L}$ on $\mathcal{E}$ by

$$\mathcal{L}(u) = \frac{1}{2}\|u - u^*\|^2. \tag{5.7}$$

From (3.2) we have $\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) = u$ and using (3.3)-(3.5):

$$\begin{aligned}\frac{d\mathcal{L}}{dt} =& \Big\langle u - u^*, \frac{du}{dt}\Big\rangle \\ =& \nu\Big\langle u - u^*, \Big\{\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) - u\Big\}\Big\rangle \\ =& -\nu\|u - u^*\|^2 + \nu\Big\langle u - u^*, \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u) - \mu\mathcal{F}(\mathcal{B}(u), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u))\big) \\ & - \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}\big(\Phi(u^*) - \mu\mathcal{F}(\mathcal{B}(u^*), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u^*))\big)\Big\rangle \\ \leq& -\nu\|u - u^*\|^2 + \nu\Lambda\|u - u^*\|^2.\end{aligned} \tag{5.8}$$

Thus,

$$\frac{d}{dt}\frac{1}{2}\|u - u^*\|^2 = -\nu(1-\Lambda)\|u(t) - u^*\|^2, \tag{5.9}$$

where

$$\Lambda = \frac{1}{\eta}\left(\sigma^p_\Phi + D_p\mu^p(\omega_2\psi_1 + \omega_1)^p - p\mu((\delta+\gamma)\sigma^p_\Phi)\right)^{\frac{1}{p}}.$$

Thus,

$$\|u - u^*\| \leq \|u_0 - u^*\| \exp\{-\nu(1-\Lambda)(t-t_0)\}. \tag{5.10}$$

Under Theorem 3.2 assumptions, $1 - \Lambda > 0$. Hence the solution trajectory of GRCTS (5.1) converges globally and exponentially to the unique solution of problem (3.1). □

6. Computational Illustration

Example 6.1. Let $\mathcal{E} = \mathbb{R}$ and consider single-valued mappings $\Phi, \mathcal{B}, \mathcal{N} : \mathcal{E} \to \mathcal{E}$ and the multivalued mapping $\mathcal{T} : \mathcal{E} \to 2^{\mathcal{E}}$ defined by

$$\Phi(u) = \frac{u}{2}, \qquad \mathcal{B}(u) = \left(1 - \frac{4\theta}{3}\right)u, \qquad \mathcal{N}(u) = -\frac{u}{3},$$

and

$$\mathcal{T}(u) = \Big\{\frac{5}{4}u\Big\},$$

where $\theta > 0$ is a constant.

The mapping $\mathcal{N}$ is nonexpansive. Fix step-size $\mu = 1$.

The corresponding resolvent and Cayley mappings are:

$$\mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(u) = \frac{4u}{7}, \qquad \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(u) = \frac{9}{14}u.$$

Let $\mathcal{F} : \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ be defined by

$$\mathcal{F}(u, v) = \frac{u + v}{2}.$$

Assume $\mathcal{F}$ is $2\big(1 - \frac{4\theta}{3}\big)$-strongly accretive with respect to Φ in the first variable and $\frac{11}{7}$-strongly accretive with respect to Φ in the second variable.

Using Algorithm , for initial point $y_0 \in \mathcal{E}$, define

$$s_n = \mathfrak{P}^{\mathcal{T}}_{\Phi,\mu}(y_n) = \frac{4}{7} y_n,$$

and

$$y_{n+1} = \Phi(s_n) - \mu\mathcal{F}\big(\mathcal{B}(s_n), \mathfrak{K}^{\mathcal{T}}_{\Phi,\mu}(s_n)\big) + \varepsilon_n.$$

Substituting explicit expressions:

$$\begin{aligned} y_{n+1} &= \frac{s_n}{2} - \frac{1}{2}\left[\left(1 - \frac{4\theta}{3}\right) s_n + \frac{9}{14} s_n\right] + \varepsilon_n \\ &= \Big(\frac{56\theta - 27}{84}\Big) s_n + \varepsilon_n. \end{aligned}$$

Using $s_n = \frac{4}{7} y_n$, we obtain the recurrence:

$$y_{n+1} = \frac{4(56\theta - 27)}{588}\, y_n + \varepsilon_n.$$

Thus, for the given mappings and parameters, the sequence $\{y_n\}$ follows a linear recurrence depending on θ, the resolvent parameter, and error term $\{\varepsilon_n\}$.

Since, $0 < \theta < 1$, define the contraction constant

$$\varrho := \left|\frac{4(56\theta - 27)}{588}\right|.$$

Therefore,

$$\varrho = \left|\frac{56\theta - 27}{147}\right| < \frac{29}{147} < 1.$$

then the iteration mapping is a strict contraction on $\mathbb{R}$.

Moreover, if the error sequence $\{\varepsilon_n\}$ satisfies

$$\lim_{n\to\infty} \varepsilon_n = 0 \quad \text{and} \quad \sum_{n=1}^{\infty} \|\varepsilon_n - \varepsilon_{n-1}\| \rho^{n-1} < \infty \quad \forall \rho \in (0, 1),$$

then $\{y_n\}$ converges strongly to the unique fixed point

$$y^* = 0,$$

which is the unique solution of the associated variational inclusion.

For $0 < \theta < 1$, all hypotheses of Theorems 3.2 and 4.1 are satisfied. Consequently, the sequence generated by Algorithm converges strongly to the unique solution $u^* = 0$ of the variational inclusion problem. To demonstrate the effectiveness of Algorithm , we present numerical results for different values of the averaging parameter θ. The results illustrate the convergence behavior predicted by the theoretical analysis.

TABLE 1. Convergence behavior of $|y_n|$ for different values of θ

Iteration	$\theta = 0.2$	$\theta = 0.9$
0	1	1
1	0.097483	0.16918
3	0.0035657	0.010954
6	0.0019501	0.002858
9	0.00079223	0.0011555
12	0.0003221	0.00046978
15	0.00013096	0.000191
17	$7.187e^{-5}$	0.00010482
19	$3.9443e^{-5}$	$5.7527e^{-5}$
20	$2.922e^{-5}$	$4.2617e^{-5}$

Table 1 reports iterations required to reach prescribed tolerance for different θ values. Smaller θ values yield faster convergence, consistent with the contraction estimate from Theorems 3.2 and 4.1.

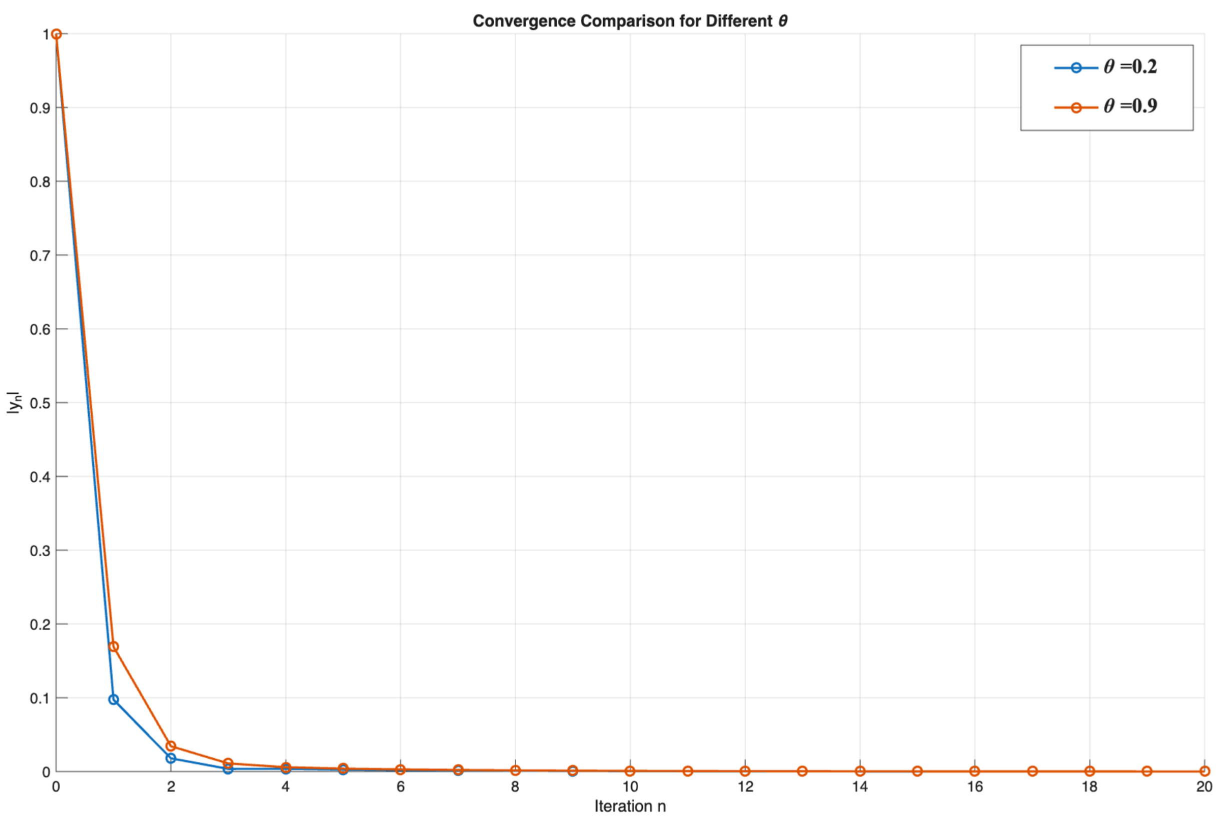

FIGURE 1. Comparison of convergence of Algorithm for different values of θ

Figure 1 illustrates convergence behavior for different θ values. Smaller θ produces smaller contraction constants and faster convergence. In particular, $\theta = 0.2$ converges significantly faster than $\theta = 0.9$, consistent with theoretical analysis.

7. Conclusion

This work develops graph convergence for Φ-accretive mappings in p-uniformly smooth Banach spaces and establishes several equivalence results connecting graph convergence with convergence of resolvent and Cayley mappings for sequences of such mappings. Based on these results, a perturbed iterative procedure was proposed for solving variational inclusions involving Φ-accretive mappings. Under appropriate conditions, solution existence and convergence of the iterative sequence were rigorously established. The study was further extended to include a generalized resolvent continuous-time system, providing a continuous-time perspective for analyzing solution trajectories and their stability. A computational illustration validated the theoretical results, confirming that the iterative procedure converges to the solution of the variational inclusion and demonstrating the utility of the generalized resolvent approach in computations.

Statements and Declarations

The authors declare no conflict of interest.

Acknowledgments

All authors have contributed equally to the work, and they have read and approved the final version. There is no funding for this research.

Data Availability Statement: Data is contained within the article.

References

[1] R. Ahmad, M. Akram, and M. Dilshad. Graph convergence for the $H(\cdot,\cdot)$-co-accretive mapping with an application. *Bulletin of the Malaysian Mathematical Sciences Society*, 38(4):1481-1506, 2015.

[2] M. Asad and M. Dilshad. On some generalized split problems and their solutions. *Acta et Commentationes Universitatis Tartuensis de Mathematica*, 29(2):259-281, 2025.

[3] A. Attouch. *Variational Convergence for Functions and Operators*. Applicable Mathematics Series, Pitman, Boston, 1984.

[4] J. Balooee and J. C. Yao. An algorithmic approach for a class of set-valued variational inclusion problems. *Filomat*, 37(13):4395-4424, 2023.

[5] Y.-P. Fang and N.-J. Huang. H-accretive operators and resolvent operator technique for solving variational inclusions in Banach spaces. *Applied Mathematics Letters*, 17(6):647-653, 2004.

[6] D. Filali, M. Dilshad, and M. Akram. Generalized variational inclusion: graph convergence and dynamical system approach. *AIMS Mathematics*, 9(9):24525-24545, 2024.

[7] D. Filali, M. Dilshad, M. Akram, M. F. Khan, and S. S. Irfan. Hybrid inertial viscosity-type forward–backward splitting algorithms for variational inclusion problems. *AIMS Mathematics*, 10:28829-28860, 2025.

[8] G. N. Ogwo, C. Izuchukwu, Y. Shehu, and X. Zhao. A new version of forward-backward-forward splitting algorithm for monotone inclusions. *Computational and Applied Mathematics*, 44:Article ID 363, 2025.

[9] B. Tan and L. Li. Strong convergence of inertial Mann algorithms for solving hierarchical fixed point problems. *Journal of Nonlinear and Variational Analysis*, 4:337-355, 2020.

[10] S. Takahashi, W. Takahashi, and M. Toyoda. Strong convergence theorems for maximal monotone operators with nonlinear mappings in Hilbert spaces. *Journal of Optimization Theory and Applications*, 147:27-41, 2010.

[11] M. F. Khan, S. S. Irfan, and I. Ahmad. Convergence analysis for system of Cayley generalized variational inclusion on q-uniformly Banach space. *Axioms*, 14(5):Article ID 361, 2025.

[12] X. Li and N.-J. Huang. Graph convergence for the $H(\cdot,\cdot)$-accretive operator in Banach spaces with an application. *Applied Mathematics and Computation*, 217(22):9053-9061, 2011.

[13] M. H. Rahaman, M. Islam, S. S. Irfan, J. C. Yao, and X. Zhao. Inertial subgradient splitting projection methods for solving equilibrium problems and applications. *Numerical Algorithms*, 98:1-35, 2025.

[14] N.-J. Huang and Y.-P. Fang. Generalized m-accretive mappings in Banach spaces. *Journal of Sichuan University (Natural Science Edition)*, 38(4):591-592, 2001.

[15] A. K. Rajpoot, J. Iqbal, R. Ahmad, and C. F. Wen. Variational inclusion problem involving averaged operator and its corresponding resolvent-type equation. *Journal of Nonlinear Convex Analysis.*, 25(10):2457-2471, 2024.

[16] M. K. Tam. Algorithms based on unions of nonexpansive maps. *Optimization Letters*, 12(5):1019-1027, 2018.

[17] Y. Cao, V. K. Khan, M. Sarfaraz, A. Arbi, and M. K. Ahmad. Second-order dynamical system to solve the sum of monotone operators via the KM algorithm. *Alexandria Engineering Journal*, 123:341-345, 2025.

[18] J.-B. Baillon, R. E. Bruck, and S. Reich. On the asymptotic behaviour of nonexpansive mappings and semigroups in Banach spaces. *Houston Journal of Mathematics*, 4:1-9, 1978.

[19] F. E. Browder. Nonlinear mappings of nonexpansive and accretive-type in Banach spaces. *Bulletin of the American Mathematical Society*, 73:875-882, 1967.

[20] F. O. Nwawuru, O. K. Narain, M. Dilshad, and J. N. Ezeora. Splitting method involving two-step inertial for solving inclusion and fixed point problems with applications. *Fixed Point Theory and Algorithms for Sciences and Engineering*, 1:Article ID 8, 2025.