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OPTIMIZING A SUPPLY-CHAIN INVENTORY MODEL HAVING PRICE AND STOCK-DEPENDENT DEMAND UNDER SUPPLIER CREDITS AND CASH DISCOUNT POLICY

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Dedicated to Professor Hari Mohan Srivastava on the Occasion of His 85th Birthday

Abstract. Traditional Economic Order Quantity (EOQ) models assume that payment for goods is made immediately upon delivery. However, suppliers often offer two key incentives simultaneously: (1) a payment delay to captivate new customers and increase sales, and (2) a cash discount to encourage prompt disbursement and diminish credit costs. This paper develops an optimal supply-chain inventory model for deteriorating goods with price and stock-dependent demand, incorporating a constant deterioration rate, under both trade credit and cash discount policies. The model assumes no shortages and considers the salvage value of deteriorated units. Our objective is to minimize costs in a scenario where the supplier provides both a financial discount and a permissible payment delay. We formulate a mathematical model and propose a solution approach in both crisp and fuzzy contexts. Numerical examples, analyzed using Mathematica 13.0.1 software, validate our findings and demonstrate the convexity of the overall cost function. Additionally, a sensitivity analysis of key parameters is conducted to provide valuable managerial insights for inventory managers.

Keywords. Price-stock dependent demand, Salvage value, Cash discount, Delay in payments, Graded mean integration representation method, Pentagonal fuzzy numbers.

1. Introduction

Current corporate policies are based on competitive marketing techniques. Every businessman or vendor has significant challenges to remain competitive. In today's competitive business world, a strong vendor-buyer relationship is critical to the success of any corporate organization. Although both buyer and vendor can support the business by establishing a relationship, the business climate requires an inventive attitude of collaboration between buyer and vendor. To attract clients and convert potential customers into repeat purchasers, business owners employ a variety of discount techniques, such as price discounts, quantity discounts, trade discounts, seasonal discounts, discounts for advance payment, and so on. Traditionally, while developing models for various inventory systems, most academics assume that the retailer's finances are sufficient and must be paid to the supplier as soon as the goods arrive. However, this assumption is not valid in the real world of business. In general, the supplier will often grant the store a trade credit period to settle their debt. During the permitted trade credit period, no interest is imposed; thereafter, interest will be charged on the unpaid amount. So, disbursing later is favorable because it lowers the stock's holding cost in the long run. This policy serves as an incentive for retailers because it allows them to sell things, accrue income, and earn interest on their capital during this time. On the other hand, this credit policy helps the provider attract more clients and improve sales. Thus, trade credit is critical to inventory management for both suppliers and retailers.

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In inventory management theory, consumer demand is a crucial element of every inventory model. The nature of demand is influenced by various factors, including selling price, stock availability, time, product quality, environmental considerations, and inherent uncertainties. As a result, researchers and decision-makers often view demand rates as a function of price, inventory levels, or a combination of factors such as selling price, stock levels, time, or environmental attributes. Deterioration is common in inventory systems dealing with perishable products like fruits, vegetables, and pharmaceuticals, leading to significant losses in both quality and quantity. To boost market demand, businesses employ various policies, including trade credit, quantity discounts, and cash discounts, each designed to enhance demand in different ways. Many inventory control studies assume a constant demand rate throughout the inventory cycle, but in reality, demand rates are influenced by factors such as selling price and product availability.

Supply chain modeling involves various processes that businesses have used for decades to establish well-organized supply chains. Different businesses have distinct supply chain requirements, with some needing to remain agile to respond to unpredictable demands. Inventory is a crucial and visible aspect of business operations, where goods are stored as spare parts, raw materials, and both partially and fully finished products. Research on inventory modeling focuses on providing decision-making tools to enhance the efficiency of inventory systems. However, designing an effective inventory control mechanism is challenging. Numerous supply chain inventory models have been developed to address the complexities faced by supply chain participants, incorporating innovative strategies for managing different aspects of the supply chain. Despite focusing on these challenges, much of the research fails to account for the uncertainties that are a fundamental part of the modern business world. Addressing both complexities and uncertainties is vital for the future of supply chain management. The supply chain management process is shown in FIGURE 1, sourced from the internet.

Traditionally, the standard Economic Order Quantity (EOQ) model assumes immediate payment for goods upon receipt. However, in practice, suppliers often provide a fixed credit period, allowing retailers to clear their accounts and thereby promoting demand. During this credit period, retailers can sell the products, accumulate revenue, and earn interest. Goyal [6] first introduced an EOQ inventory model incorporating a payment delay. Subsequently, Aggarwal and Jaggi [1] expanded this framework to account for decaying items. Later, Jamal *et al.* [8] further extended the framework to include shortages. Liao *et al.* [16] devised an inventory framework with stock-dependent consumption rates that also accommodates payment delays. Ouyang *et al.* [26] introduced an EOQ model incorporating both cash discounts and payment delay policies. This was followed by Ouyang *et al.* [27], who proposed an EOQ model for deteriorating items under trade credit finance. Shah and Mishra [39] then developed an EOQ model for deteriorating commodities with stock-dependent demand under supplier credits. Building on this, Rastogi *et al.* [38] suggested an EOQ framework considering variable holding costs with trade credit and cash discount policies. Chung *et al.* [4] created an inventory model for deteriorating items with conditional cash discount facilities. Lastly, Shah and Naik [40] developed a model for deteriorating inventory with quadratic demand under trade credit and cash discount policies. There are some other works on inventories under cash discount policy by [7, 14, 17, 30, 36, 44].

Typically, demand rates are assumed to be either constant or time-dependent, without considering the impact of stock levels or selling prices. However, research indicates that increasing shelf space for an item can boost its sales due to enhanced visibility and attractiveness, which attracts more customers. Conversely, low stock levels might suggest that items are not fresh or desirable, thereby affecting their demand rates. Datta and Paul [5] developed a finite-horizon inventory model in which demand rates depend on both supply and pricing. Teng and Chang [43] proposed an Economic Production Quantity (EPQ) inventory model for deteriorating products, incorporating demand based on selling price and stock levels. Later, Soni [42] introduced an inventory model for non-instantaneously deteriorating items, considering price and stock-sensitive demand under acceptable payment delays. Pal *et al.* [29]

FIGURE 1. Supply Chain Management Process

formulated a model addressing price and stock-dependent demand rates in the context of inflation. Following this, Chowdhury *et al.* [3] developed an inventory model for degrading items with price-sensitive demand. Mishra [18] established a waiting-time deterministic inventory model for decaying products with time-dependent demand. Mishra *et al.* [20] further devised a model for controllable degradation rates with shortages, accounting for price and stock-dependent demand. Additional works on inventory models incorporating stock and price-dependent demand include those by [19, 21, 31, 32, 37, 41].

In most real-world scenarios, accurate data is insufficient for a mathematical model. Usually, human choices, including inclinations, are vague and cannot be evaluated in terms of inclinations with specific numerical information. Fuzzy set theory is an incredible tool for illustrating the type of instability associated with vagueness, imprecision, and a lack of data regarding a certain topic at hand. L. A. Zadeh [45] introduced fuzzy set theory as an extension of traditional sets (crisp sets or classical sets). Eventually, Lee and Yao [15] addressed the impreciseness in production quantity and demand in their inventory model by utilizing fuzzy ideas. In this context, Kumar and Paikray [12] developed a cost optimization inventory framework for degrading items with a trapezoidal type demand rate in both fuzzy and crisp circumstances. Kumar *et al.* [13] suggested a retailer's inventory framework for degrading items in both crisp and fuzzy scenarios. Similarly, Nayak *et al.* [23] developed an EOQ framework for decaying items with time-sensitive demand under partially backlogged shortages in fuzzy contexts. Padhy *et al.* [28] also introduced an EOQ framework that addresses both improvements and degradation in fuzzy contexts. Additionally, one may refer to the works of [9, 10, 11, 22, 24, 25, 33, 34, 35] for several constraints in both crisp and fuzzy contexts.

Based on the aforementioned discussions, it is evident that none of the existing inventory models have addressed price and stock-dependent demand with a cash discount policy under cost imprecision. However, inventory models incorporating all these constraints are highly relevant to many businesses in real-market scenarios. Consequently, this gap in the literature has led us to develop an optimal inventory framework that includes price and stock-sensitive demand, permissible payment delays, and cash discount policies, all while considering cost imprecision. Our objective is to minimize the total cost. To achieve this, we have constructed a computational algorithm and provided numerical examples to illustrate the theoretical results under various conditions. Additionally, we offer managerial insights and conduct a sensitivity analysis to benefit inventory managers.

The framework of our current investigation is as follows: Section 1 presents the introduction and outlines the motivation for our proposed study. Section 2 provides the notations and assumptions to frame the problem. Section 3 details the framework formulation in crisp contexts. Section 4 explains the computational solution algorithm for the problem. Section 5 discusses the framework formulation under fuzzy scenarios. Section 6 provides numerical illustrations to demonstrate the applicability of the study. Section 7 presents the sensitivity analysis and managerial insights derived from the model. Section 8 summarizes the findings and conclusions of our study.

2. Notations and Assumptions

The following notations and assumptions provide the basis for the mathematical formulation of the proposed inventory problem.

2.1. Notations.

TABLE 1 summarizes the terminology used in this paper's mathematical model development.

Note that the functions prefixed with the notation $\widetilde{\ }$ represent fuzzy-valued functions with imprecise parameters.

2.2. Assumptions.

(i) The inventory system manages a single item.
(ii) In crisp context, the associated costs are predictable, whereas in fuzzy context, they are characterized by imprecision.
(iii) The replenishment process occurs instantly.
(iv) The demand rate pursues the price and stock-dependent as $D(t) = \mathcal{A} + \zeta I(t) - \mathcal{P}; \mathcal{A} > 0, 0 < \zeta \ll 1$.
(v) The time horizon is infinite, and there is no lead time.
(vi) Shortages are prohibited.
(vii) The revenue from sales is deposited into an interest-bearing account while it remains unpaid. At the end of the term, the customer retains all profits, settles the payment for all sold units, and begins paying interest on the inventory that remains in stock.
(viii) Pentagonal fuzzy numbers are used to quantify imprecise costs in fuzzy scenarios.

3. Model Formulation

The goal of the model is to identify the optimal order quantity that minimizes the total cost. During the interval $[0, T]$, the inventory level decreases primarily to meet demand and secondarily due to deterioration. Consequently, the inventory level at any time t is illustrated in FIGURE 2 and described mathematically as follows.

TABLE 1. Notations

Parameters	
O_C	unit ordering cost
H_C	unit holding cost
P_C	unit purchase cost
$\mathcal{Q}$	order quantities per cycle
$\mathcal{P}$	selling price
ν	deterioration rate (constant; $0 \leq \nu < 1$)
I_E	interest earned rate
I_C	interest paid rate
$\mathcal{R}$	discount rate; $0 < \mathcal{R} < 1$
$\mathcal{N}$	discount time frame
$\mathcal{M}$	allowable delay duration; $\mathcal{M} > \mathcal{N}$
η	salvage value; $0 \leq \eta < 1$
$\mathcal{A}$	a positive constant
ζ	stock-dependent consumption rate
Π_D	number of deteriorating items
Decision variables	
T	cycle length
Functions	
$D(t)$	demand rate
$I(t)$	in-stock inventory in $[0, T]$
$TC(T)$	per unit overall cost
$\widetilde{TC}(T)$	per unit overall cost fuzzy environment
$GTC(T)$	defuzzified total cost function

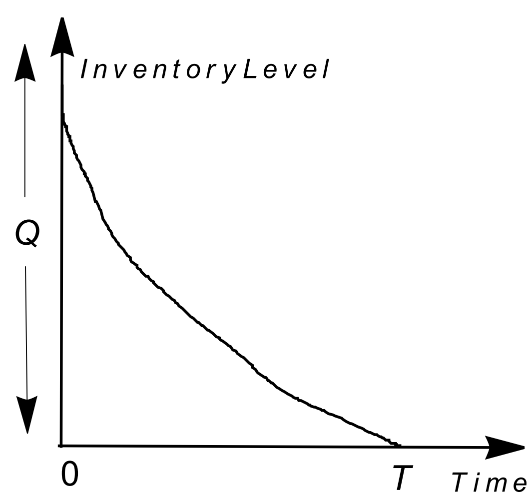


FIGURE 2. Stock level at time t

Let $I(t)$ denote the in-stock inventory at time $t \geq 0$ within the interval $[0, T]$. Thus, the differential equation under the boundary condition $I(T) = 0$ is given by

$$\frac{dI(t)}{dt} + \nu I(t) = -\left(\mathcal{A} + \zeta I(t) - \mathcal{P}\right), \quad 0 < t \leq T. \tag{3.1}$$

Solving equation (3.1) using the boundary condition, we obtain

$$I(t) = \frac{\mathcal{A}-\mathcal{P}}{\zeta+\nu}\left\{e^{(\zeta+\nu)(T-t)} - 1\right\}. \tag{3.2}$$

The order quantity for each replenishment cycle is defined as

$$\mathcal{Q} = I(t=0) = \frac{\mathcal{A}-\mathcal{P}}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1\right\}.$$

The total demand over one cycle is expressed as

$$\prod\left(\mathcal{Q}(T)\right)T = \left(\mathcal{A}+\zeta I(T)-\mathcal{P}\right)T = \left(\mathcal{A}-\mathcal{P}\right)T.$$

During a replenishment cycle, the quantity of decaying products is given by

$$\Pi_D = \mathcal{Q} - \prod\left(\mathcal{Q}(T)\right)T = \frac{\mathcal{A}-\mathcal{P}}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T\left(\zeta+\nu\right)\right\}.$$

Next, in view of calculating the overall cost, the various associated factors are obtained as follows.

Ordering Cost.

$$OC = O_C. \tag{3.3}$$

Holding Cost.

$$HC = H_C \times \int_0^T I(t)dt = \frac{H_C\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)T} - 1 - T\left(\zeta+\nu\right)\right\}. \tag{3.4}$$

Deterioration Cost.

$$DC = P_C \times \Pi_D = \frac{P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T\left(\zeta+\nu\right)\right\}. \tag{3.5}$$

Salvage Value.

$$SV = \eta \times DC = \frac{\eta P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T\left(\zeta+\nu\right)\right\}. \tag{3.6}$$

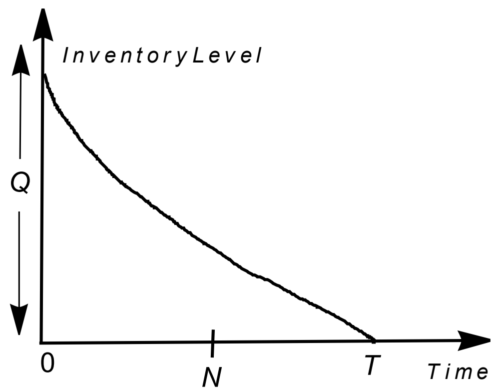
Purchase Cost.

$$PC = P_C \times \mathcal{Q} = \frac{P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1\right\}. \tag{3.7}$$

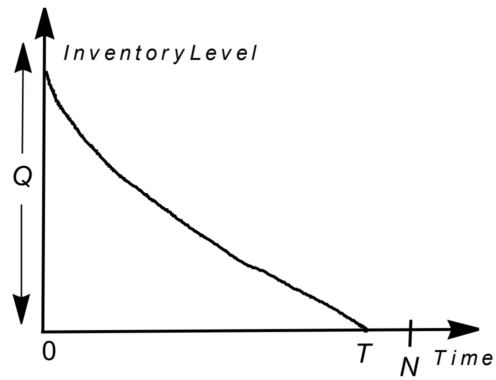
Considering the cash discount, interest charges, and interest earned, we have four scenarios based on the retailer's two payment options: paying at $\mathcal{N}$ or at $\mathcal{M}$.

- Scenario 1: $\mathcal{N} \leq T$.
- Scenario 2: $\mathcal{N} > T$.
- Scenario 3: $\mathcal{M} \leq T$.
- Scenario 4: $\mathcal{M} > T$.

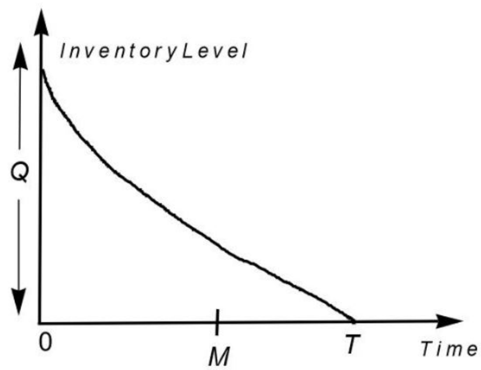
Now, we show all four scenarios graphically in FIGURE 3. Additionally, the specifics of each scenarios are provided below.



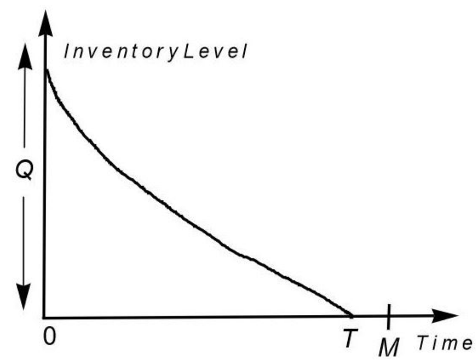
(A) Scenario 1: $\mathcal{N} \leq T$



(B) Scenario 2: $\mathcal{N} > T$



(C) Scenario 3: $\mathcal{M} \leq T$



(D) Scenario 4: $\mathcal{M} > T$

FIGURE 3. Stock level at time T

Scenario 1. $\mathcal{N} \leq T$

Since the payment is made on time ($\mathcal{N}$), the customer benefits from a price discount, resulting in a savings of ($\mathcal{R}P_C\mathcal{Q}$) per cycle. Hence, the cash discount is given by

$$CD = \mathcal{R}P_C\mathcal{Q} = \frac{\mathcal{R}P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1\right\}. \tag{3.8}$$

Next, the items in stocks have to be financed after time ($\mathcal{N}$). So the interest payable is given by

$$IP_1 = P_C I_C \times \int_{\mathcal{N}}^{T} I(t)dt = \frac{P_C I_C\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)(T-\mathcal{N})}-1-\left(T-\mathcal{N}\right)\left(\zeta+\nu\right)\right\}. \tag{3.9}$$

In the period $[0,\mathcal{N}]$, the customer sells goods and deposits the revenue into an account. Consequently, the interest earned is expressed as

$$\begin{aligned} IE_1 &= \mathcal{P}I_E \times \int_0^{\mathcal{N}} tD(t)dt \\ &= \mathcal{P}I_E\left\{\frac{\left(\mathcal{A}-\mathcal{P}\right)\nu\mathcal{N}^2}{2\left(\zeta+\nu\right)}+\frac{\zeta\left(\mathcal{A}-\mathcal{P}\right)e^{(\zeta+\nu)T}}{\left(\zeta+\nu\right)^3}\left\{1-\mathcal{N}\left(\zeta+\nu\right)e^{-\mathcal{N}(\zeta+\nu)}-e^{-\mathcal{N}(\zeta+\nu)}\right\}\right\}. \end{aligned} \tag{3.10}$$

The overall cost per unit of time is defined as

$$TC_1(T) = \frac{\mathbb{W}_1}{T}, \tag{3.11}$$

where

$$\begin{aligned}\mathbb{W}_1 &= OC + HC + DC + PC + IP_1 - CD - SV - IE_1 \\ &= O_C + \frac{H_C(\mathcal{A}-\mathcal{P})}{(\zeta+\nu)^2}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} + \frac{P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} \\ &\quad + \frac{P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1\right\} + \frac{P_C I_C(\mathcal{A}-\mathcal{P})}{(\zeta+\nu)^2}\left\{e^{(\zeta+\nu)(T-\mathcal{N})} - 1 - (T-\mathcal{N})(\zeta+\nu)\right\} \\ &\quad - \frac{\mathcal{R}P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1\right\} - \frac{\eta P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} \\ &\quad - \mathcal{P}I_E\left\{\frac{(\mathcal{A}-\mathcal{P})\nu\mathcal{N}^2}{2(\zeta+\nu)} + \frac{\zeta(\mathcal{A}-\mathcal{P})e^{(\zeta+\nu)T}}{(\zeta+\nu)^3}\left\{1 - \mathcal{N}(\zeta+\nu)e^{-\mathcal{N}(\zeta+\nu)} - e^{-\mathcal{N}(\zeta+\nu)}\right\}\right\}.\end{aligned}$$

Scenario 2. $\mathcal{N} > T$

In this scenario, the consumer sells all units at time T and makes the full payment to the provider at time $\mathcal{N}$. As a result, no interest is incurred, and the cash discount remains the same as in Scenario 1. Thus, the interest earned is expressed as

$$\begin{aligned}IE_2 &= \mathcal{P}I_E \times \left\{\int_0^T tD(t)dt + (\mathcal{A}-\mathcal{P})T(\mathcal{N}-T)\right\} \\ &= \mathcal{P}I_E\left\{\frac{(\mathcal{A}-\mathcal{P})\nu T^2}{2(\zeta+\nu)} + \frac{\zeta(\mathcal{A}-\mathcal{P})}{(\zeta+\nu)^3}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} + (\mathcal{A}-\mathcal{P})T(\mathcal{N}-T)\right\}.\end{aligned} \tag{3.12}$$

The overall cost per unit of time is calculated as

$$TC_2(T) = \frac{\mathbb{W}_2}{T}, \tag{3.13}$$

where

$$\begin{aligned}\mathbb{W}_2 &= OC + HC + DC + PC + IP_2 - CD - SV - IE_2 \\ &= O_C + \frac{H_C(\mathcal{A}-\mathcal{P})}{(\zeta+\nu)^2}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} + \frac{P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} \\ &\quad + \frac{P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1\right\} - \frac{\mathcal{R}P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1\right\} \\ &\quad - \frac{\eta P_C(\mathcal{A}-\mathcal{P})}{\zeta+\nu}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} - \mathcal{P}I_E\left\{\frac{(\mathcal{A}-\mathcal{P})\nu T^2}{2(\zeta+\nu)}\right. \\ &\quad \left. + \frac{\zeta(\mathcal{A}-\mathcal{P})}{(\zeta+\nu)^3}\left\{e^{(\zeta+\nu)T} - 1 - T(\zeta+\nu)\right\} + (\mathcal{A}-\mathcal{P})T(\mathcal{N}-T)\right\}.\end{aligned}$$

Scenario 3. $\mathcal{M} \leq T$

The payment is completed at time $\mathcal{M}$, so no cash discount applies. The interest to be paid is calculated as

$$IP_3 = P_C I_C \times \int_{\mathcal{M}}^T I(t)dt = \frac{P_C I_C(\mathcal{A}-\mathcal{P})}{(\zeta+\nu)^2}\left\{e^{(\zeta+\nu)(T-\mathcal{M})} - 1 - (T-\mathcal{M})(\zeta+\nu)\right\}. \tag{3.14}$$

The interest earned can be expressed as

$$IE_3 = \mathcal{P}I_E \times \int_0^{\mathcal{M}} tD(t)dt$$
$$= \frac{\mathcal{P}I_E\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{\frac{\nu\mathcal{M}^2}{2}+\frac{\zeta}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)T}-\mathcal{M}\left(\zeta+\nu\right)e^{(\zeta+\nu)(T-\mathcal{M})}-e^{(\zeta+\nu)(T-\mathcal{M})}\right\}\right\}. \tag{3.15}$$

The overall cost per unit of time is expressed as

$$TC_3(T) = \frac{\mathbb{W}_3}{T}, \tag{3.16}$$

where

$$\mathbb{W}_3 = OC + HC + DC + PC + IP_3 - CD - SV - IE_3$$
$$= O_C + \frac{H_C\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}+\frac{P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}$$
$$+\frac{P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1\right\}+\frac{P_CI_C\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)(T-\mathcal{M})}-1-\left(T-\mathcal{M}\right)\left(\zeta+\nu\right)\right\}$$
$$-\frac{\eta P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}-\frac{\mathcal{P}I_E\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{\frac{\nu\mathcal{M}^2}{2}\right.$$
$$\left.+\frac{\zeta}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)T}-\mathcal{M}\left(\zeta+\nu\right)e^{(\zeta+\nu)(T-\mathcal{M})}-e^{(\zeta+\nu)(T-\mathcal{M})}\right\}\right\}\Bigg\}.$$

Scenario 4. $\mathcal{M} > T$

In this scenario, no interest is imposed. The earned interest is expressed as

$$IE_4 = \mathcal{P}I_E \times \left\{\int_0^T tD(t)dt + \left(\mathcal{A}-\mathcal{P}\right)T\left(\mathcal{M}-T\right)\right\}$$
$$= \mathcal{P}I_E\left\{\frac{\left(\mathcal{A}-\mathcal{P}\right)\nu T^2}{2\left(\zeta+\nu\right)}+\frac{\zeta\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^3}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}+\left(\mathcal{A}-\mathcal{P}\right)T\left(\mathcal{M}-T\right)\right\}. \tag{3.17}$$

The unit overall cost is expressed by

$$TC_4(T) = \frac{\mathbb{W}_4}{T}, \tag{3.18}$$

where

$$\mathbb{W}_4 = OC + HC + DC + PC + IP_4 - CD - SV - IE_4$$
$$= O_C + \frac{H_C\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^2}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}+\frac{P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}$$
$$+\frac{P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1\right\}-\frac{\eta P_C\left(\mathcal{A}-\mathcal{P}\right)}{\zeta+\nu}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}$$
$$-\mathcal{P}I_E\left\{\frac{\left(\mathcal{A}-\mathcal{P}\right)\nu T^2}{2\left(\zeta+\nu\right)}+\frac{\zeta\left(\mathcal{A}-\mathcal{P}\right)}{\left(\zeta+\nu\right)^3}\left\{e^{(\zeta+\nu)T}-1-T\left(\zeta+\nu\right)\right\}+\left(\mathcal{A}-\mathcal{P}\right)T\left(\mathcal{M}-T\right)\right\}\Bigg\}.$$

Hence, in the four scenarios mentioned above, the overall cost is obtained as

$$TC_i(T) = \begin{cases} TC_1(T), & (\mathcal{N} \leq T) \\ TC_2(T), & (\mathcal{N} > T) \\ TC_3(T), & (\mathcal{M} \leq T) \\ TC_4(T), & (\mathcal{M} > T) \end{cases} \tag{3.19}$$

for $i = 1, 2, 3, 4$.

4. Computational Algorithm

The traditional optimum strategy is utilized to solve the problem. The primary objective is to minimize the overall cost function $TC_i(T)$. Here, the steps to the solution are as follows to verify that the decision parameter is optimal.

Step 1 Initialize the inventory parameters; $O_C, H_C, P_C, \mathcal{P}, \nu, I_E, I_C, \mathcal{R}, \mathcal{N}, \mathcal{M}, \eta, \mathcal{A}$, and ζ.
Step 2 Determine $TC_1(T)$ from equation (3.11).
Step 3 Find $\frac{\partial TC_1(T)}{\partial T}$.
Step 4 Resolve the equation $\frac{\partial TC_1(T)}{\partial T} = 0$ for T.
Step 5 Select the solution from Step 4.
Step 6 Find $\frac{\partial^2 TC_1(T)}{\partial T^2}$.
Step 7 Check if $\frac{\partial^2 TC_1(T)}{\partial T^2} > 0$, then this solution is optimal (minimum).
Step 8 Otherwise proceed to Step 5.

Note that, similar results can be established for the other objective functions under the consideration of equations (3.13), (3.16) and (3.18).

5. Fuzzy Model

The inventory parameters can be impractical in real-life scenarios for various reasons. For instance, fuzziness makes it challenging to define each parameter precisely. To address this imprecision, we propose a suitable framework under fuzzy conditions. Specifically, to handle the fuzzy paradigm, we consider the fuzzy parameters $\widetilde{O_C}$, $\widetilde{\mathcal{A}}$, $\widetilde{\nu}$, and $\widetilde{P_C}$, which correspond to the ordering cost (O_C), demand parameter ($\mathcal{A}$), deterioration rate (ν), and purchase cost (P_C) of our crisp framework.

The overall fuzzy cost is given by

$$\widetilde{TC_i}(T) = \begin{cases} \widetilde{TC_1}(T), & (\mathcal{N} \leq T) \\ \widetilde{TC_2}(T), & (\mathcal{N} > T) \\ \widetilde{TC_3}(T), & (\mathcal{M} \leq T) \\ \widetilde{TC_4}(T), & (\mathcal{M} > T) \end{cases} \tag{5.1}$$

for $i = 1, 2, 3, 4$.
That is,

$$\begin{aligned} \widetilde{TC_1}(T) = \frac{1}{T}\Bigg\{ & \widetilde{O_C} + \frac{H_C\left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{(\zeta+\widetilde{\nu})^2}\left\{e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta+\widetilde{\nu}\right)\right\} + \frac{\widetilde{P_C}\left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta+\widetilde{\nu}}\left\{e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta+\widetilde{\nu}\right)\right\} \\ & + \frac{\widetilde{P_C}\left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta+\widetilde{\nu}}\left\{e^{(\zeta+\widetilde{\nu})T} - 1\right\} + \frac{\widetilde{P_C} I_C\left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{(\zeta+\widetilde{\nu})^2}\left\{e^{(\zeta+\widetilde{\nu})(T-\mathcal{N})} - 1 - (T-\mathcal{N})\left(\zeta+\widetilde{\nu}\right)\right\} \\ & - \frac{\mathcal{R}\widetilde{P_C}\left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta+\widetilde{\nu}}\left\{e^{(\zeta+\widetilde{\nu})T} - 1\right\} - \frac{\eta\widetilde{P_C}\left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta+\widetilde{\nu}}\left\{e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta+\widetilde{\nu}\right)\right\} \end{aligned}$$

$$- \mathcal{P} I_E \left\{ \frac{\left(\widetilde{\mathcal{A}} - \mathcal{P}\right) \widetilde{\nu} \mathcal{N}^2}{2\left(\zeta + \widetilde{\nu}\right)} + \frac{\zeta \left(\widetilde{\mathcal{A}} - \mathcal{P}\right) e^{(\zeta+\widetilde{\nu})T}}{\left(\zeta + \widetilde{\nu}\right)^3} \left\{ 1 - \mathcal{N}\left(\zeta + \widetilde{\nu}\right) e^{-\mathcal{N}(\zeta+\widetilde{\nu})} - e^{-\mathcal{N}(\zeta+\widetilde{\nu})} \right\} \right\} \Bigg\};$$

$$\begin{aligned} \widetilde{TC}_2(T) = \frac{1}{T} \Bigg\{ & \widetilde{O_C} + \frac{H_C \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\left(\zeta + \widetilde{\nu}\right)^2} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} + \frac{\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} \\ & + \frac{\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 \right\} - \frac{\mathcal{R}\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 \right\} \\ & - \frac{\eta \widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} - \mathcal{P} I_E \Bigg\{ \frac{\left(\widetilde{\mathcal{A}} - \mathcal{P}\right) \widetilde{\nu} T^2}{2\left(\zeta + \widetilde{\nu}\right)} \\ & + \frac{\zeta \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\left(\zeta + \widetilde{\nu}\right)^3} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} + \left(\widetilde{\mathcal{A}} - \mathcal{P}\right) T\left(\mathcal{N} - T\right) \Bigg\} \Bigg\}; \end{aligned}$$

$$\begin{aligned} \widetilde{TC}_3(T) = \frac{1}{T} \Bigg\{ & \widetilde{O_C} + \frac{H_C \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\left(\zeta + \widetilde{\nu}\right)^2} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} + \frac{\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} \\ & + \frac{\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 \right\} + \frac{\widetilde{P_C} I_C \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\left(\zeta + \widetilde{\nu}\right)^2} \left\{ e^{(\zeta+\widetilde{\nu})(T-\mathcal{M})} - 1 - \left(T - \mathcal{M}\right)\left(\zeta + \widetilde{\nu}\right) \right\} \\ & - \frac{\eta \widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} - \frac{\mathcal{P} I_E \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \Bigg\{ \frac{\widetilde{\nu} \mathcal{M}^2}{2} \\ & + \frac{\zeta}{\left(\zeta + \widetilde{\nu}\right)^2} \left\{ e^{(\zeta+\widetilde{\nu})T} - \mathcal{M}\left(\zeta + \widetilde{\nu}\right) e^{(\zeta+\widetilde{\nu})(T-\mathcal{M})} - e^{(\zeta+\widetilde{\nu})(T-\mathcal{M})} \right\} \Bigg\} \Bigg\}; \end{aligned}$$

and

$$\begin{aligned} \widetilde{TC}_4(T) = \frac{1}{T} \Bigg\{ & \widetilde{O_C} + \frac{H_C \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\left(\zeta + \widetilde{\nu}\right)^2} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} + \frac{\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} \\ & + \frac{\widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 \right\} - \frac{\eta \widetilde{P_C} \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\zeta + \widetilde{\nu}} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} \\ & - \mathcal{P} I_E \Bigg\{ \frac{\left(\widetilde{\mathcal{A}} - \mathcal{P}\right) \widetilde{\nu} T^2}{2\left(\zeta + \widetilde{\nu}\right)} + \frac{\zeta \left(\widetilde{\mathcal{A}} - \mathcal{P}\right)}{\left(\zeta + \widetilde{\nu}\right)^3} \left\{ e^{(\zeta+\widetilde{\nu})T} - 1 - T\left(\zeta + \widetilde{\nu}\right) \right\} + \left(\widetilde{\mathcal{A}} - \mathcal{P}\right) T\left(\mathcal{M} - T\right) \Bigg\} \Bigg\}. \end{aligned}$$

5.1. **Defuzzification.**

The fuzzy parameters are represented using pentagonal fuzzy numbers as $\widetilde{O_C} = (O_{C1}, O_{C2}, O_{C3}, O_{C4}, O_{C5})$, $\widetilde{P_C} = (P_{C1}, P_{C2}, P_{C3}, P_{C4}, P_{C5})$, $\widetilde{\mathcal{A}} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5)$, and $\widetilde{\nu} = (\nu_1, \nu_2, \nu_3, \nu_4, \nu_5)$. Thus, following Chen and Hsieh [2], the overall fuzzy cost functions are defuzzified using the Graded Mean Integration Representation (GMIR) method, yielding the total defuzzified cost as

$$GTC_i(T) = \frac{1}{12} \left\{ \widetilde{TC}_{i1} + 3 \times \widetilde{TC}_{i2} + 4 \times \widetilde{TC}_{i3} + 3 \times \widetilde{TC}_{i4} + \widetilde{TC}_{i5} \right\} \tag{5.2}$$

for $i = 1, 2, 3, 4$. Here, $\widetilde{TC}_{ij}$ is obtained from the above equation (5.1) just by replacing the imprecise parameters in $\widetilde{TC}_i$ $(i = 1, 2, 3, 4)$ with the corresponding j^{th} pentagonal fuzzy numbers for $j = 1, 2, 3, 4, 5$.

We can find an optimal solution for the fuzzy model by following a process similar to that of the crisp model solution process.

6. Numerical Illustrations

We numerically investigated the model in both crisp and fuzzy contexts under our proposed method of solution. We utilize Mathematica 13.0.1 software to determine the optimal solution and to demonstrate the convexity of the overall cost functions, as illustrated in FIGURES (4 - 7). The data utilized in the numerical examples are hypothetical in nature.

Example 1

(a) Crisp Model
We consider the succeeding inventory constraints:
$O_C = 450$, $H_C = 2$, $P_C = 20$, $\mathcal{P} = 30$, $\nu = 0.02$, $I_E = 0.07$, $I_C = 0.15$, $\mathcal{R} = 0.12$, $\mathcal{N} = 1.2$, $\mathcal{M} = 1.7$, $\eta = 0.03$, $\mathcal{A} = 100$, and $\zeta = 0.01$.

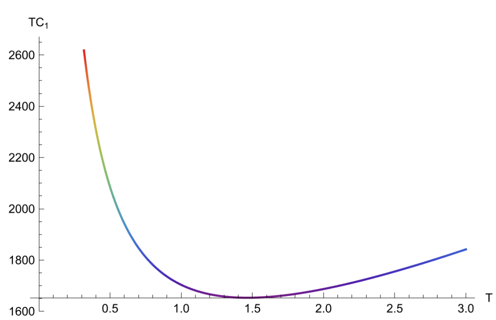
Solution
The aforementioned data gives the following ideal solutions:

$$T^* = 1.46428, \qquad \mathcal{Q}^* = 105.561, \qquad TC_1^* = 1652.97.$$

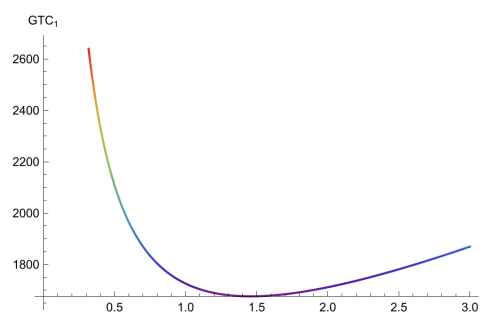
(b) Fuzzy Model
We consider the following inventory constraints:
$H_C = 2$, $\mathcal{P} = 30$, $I_E = 0.07$, $I_C = 0.15$, $\mathcal{R} = 0.12$, $\mathcal{N} = 1.2$, $\mathcal{M} = 1.7$, $\eta = 0.03$, $\zeta = 0.01$, and the corresponding fuzzy parameters are $\widetilde{O_C} = (O_{C1}, O_{C2}, O_{C3}, O_{C4}, O_{C5}) = (360, 405, 450, 495, 540)$, $\widetilde{P_C} = (P_{C1}, P_{C2}, P_{C3}, P_{C4}, P_{C5}) = (16, 18, 20, 22, 24)$, $\widetilde{\nu} = (\nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = (0.016, 0.018, 0.02, 0.022, 0.024)$, $\widetilde{\mathcal{A}} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5) = (80, 90, 100, 110, 120)$ respectively.

Solution
Taking into account the above information, we acquire the following optimal solutions:

$$T^* = 1.4572, \qquad \mathcal{Q}^* = 105.062, \qquad GTC_1^* = 1676.01.$$



(A) TC_1 vs T



(B) GTC_1 vs T

Figure 4. Convexities of total cost by employing Example 1

Example 2

(a) Crisp Model
We consider the succeeding inventory constraints:
$O_C = 550$, $H_C = 4$, $P_C = 25$, $\mathcal{P} = 50$, $\nu = 0.04$, $I_E = 0.09$, $I_C = 0.25$, $\mathcal{R} = 0.22$, $\mathcal{N} = 1.5$, $\mathcal{M} = 2$, $\eta = 0.05$, $\mathcal{A} = 200$, and $\zeta = 0.03$.

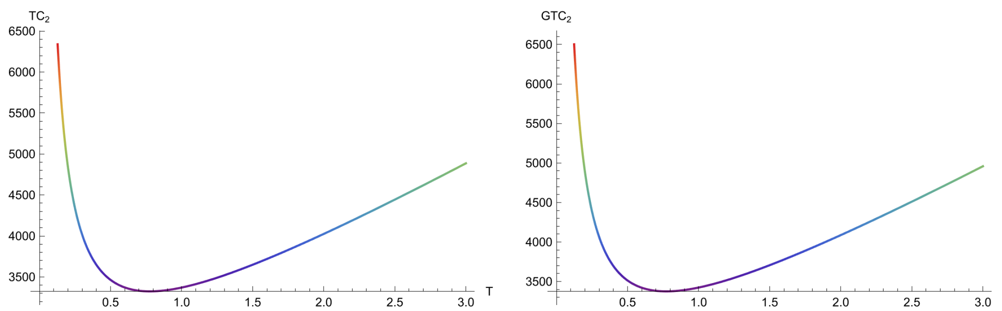
Solution
The aforementioned data gives the following ideal solutions:

$$T^* = 0.775916, \qquad \mathcal{Q}^* = 120.076, \qquad TC_2^* = 3323.71.$$

(b) Fuzzy Model
We consider the following inventory constraints:
$H_C = 4$, $\mathcal{P} = 50$, $I_E = 0.09$, $I_C = 0.25$, $\mathcal{R} = 0.22$, $\mathcal{N} = 1.5$, $\mathcal{M} = 2$, $\eta = 0.05$, $\zeta = 0.03$, and the corresponding fuzzy parameters are $\widetilde{O_C} = (O_{C1}, O_{C2}, O_{C3}, O_{C4}, O_{C5}) = (440, 495, 550, 605, 660)$, $\widetilde{P_C} = (P_{C1}, P_{C2}, P_{C3}, P_{C4}, P_{C5}) = (20, 22.5, 25, 27.5, 30)$, $\widetilde{\nu} = (\nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = (0.032, 0.036, 0.04, 0.044, 0.048)$, $\widetilde{\mathcal{A}} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5) = (160, 180, 200, 220, 240)$ respectively.

Solution
Taking into account the above information, we acquire the following optimal solutions:

$$T^* = 0.772498, \qquad \mathcal{Q}^* = 119.559, \qquad GTC_2^* = 3375.30.$$



(A) TC_2 vs T (B) GTC_2 vs T

FIGURE 5. Convexities of total cost by employing Example 2

Example 3

(a) Crisp Model
We consider the following inventory constraints:
$O_C = 500$, $H_C = 3$, $P_C = 27$, $\mathcal{P} = 40$, $\nu = 0.05$, $I_E = 0.15$, $I_C = 0.35$, $\mathcal{R} = 0.43$, $\mathcal{N} = 0.2$, $\mathcal{M} = 0.6$, $\eta = 0.07$, $\mathcal{A} = 150$, and $\zeta = 0.06$.

Solution
The aforementioned data gives the following ideal solutions:

$$T^* = 0.745266, \qquad \mathcal{Q}^* = 85.433, \qquad TC_3^* = 3860.82.$$

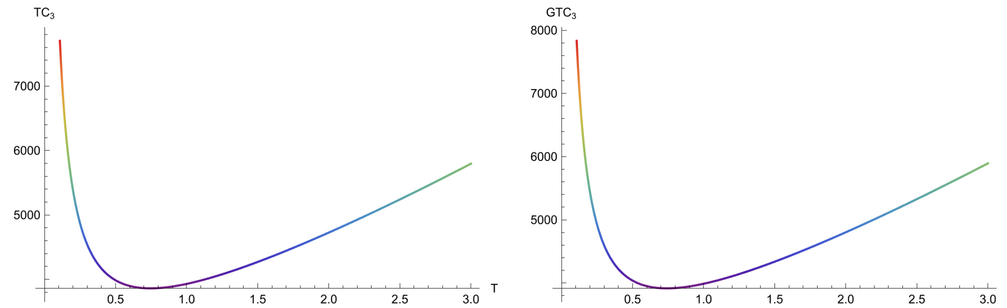
(b) Fuzzy Model
We consider the following inventory constraints:
$H_C = 3$, $\mathcal{P} = 40$, $I_E = 0.15$, $I_C = 0.35$, $\mathcal{R} = 0.43$, $\mathcal{N} = 0.2$, $\mathcal{M} = 0.6$, $\eta = 0.07$, $\zeta = 0.06$, and the corresponding fuzzy parameters are $\widetilde{O_C} = (O_{C1}, O_{C2}, O_{C3}, O_{C4}, O_{C5}) = (400, 450, 500, 550, 600)$, $\widetilde{P_C} = (P_{C1}, P_{C2}, P_{C3}, P_{C4}, P_{C5}) = (21.6, 24.3, 27, 29.7, 32.4)$, $\widetilde{\nu} = (\nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = (0.04, 0.045, 0.05, 0.055, 0.06)$, $\widetilde{\mathcal{A}} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5) = (120, 135, 150, 165, 180)$ respectively.

Solution
Taking into account the above information, we acquire the following optimal solutions:

$$T^* = 0.740795, \qquad \mathcal{Q}^* = 84.925, \qquad GTC_3^* = 3915.28.$$

Example 4

(a) Crisp Model
We consider the following inventory constraints:
$O_C = 600$, $H_C = 5$, $P_C = 30$, $\mathcal{P} = 35$, $\nu = 0.07$, $I_E = 0.25$, $I_C = 0.45$, $\mathcal{R} = 0.62$, $\mathcal{N} = 1.1$, $\mathcal{M} = 1.5$, $\eta = 0.09$, $\mathcal{A} = 120$, and $\zeta = 0.08$.



(A) TC_3 vs T (B) GTC_3 vs T

FIGURE 6. Convexities of total cost by employing Example 3

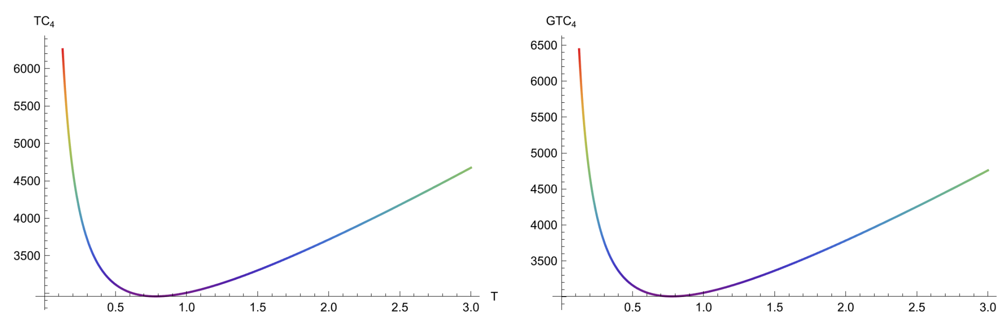
Solution
The aforementioned data gives the following ideal solutions:

$$T^* = 0.740795, \qquad \mathcal{Q}^* = 70.543, \qquad TC_4^* = 2956.23.$$

(b) Fuzzy Model
We consider the following inventory constraints:
$H_C = 5$, $\mathcal{P} = 35$, $I_E = 0.25$, $I_C = 0.45$, $\mathcal{R} = 0.62$, $\mathcal{N} = 1.1$, $\mathcal{M} = 1.5$, $\eta = 0.09$, $\zeta = 0.08$, and the corresponding fuzzy parameters are $\widetilde{O_C} = (O_{C1}, O_{C2}, O_{C3}, O_{C4}, O_{C5}) = (480, 540, 600, 660, 720)$, $\widetilde{P_C} = (P_{C1}, P_{C2}, P_{C3}, P_{C4}, P_{C5}) = (24, 27, 30, 33, 36)$, $\widetilde{\nu} = (\nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = (0.056, 0.063, 0.07, 0.077, 0.084)$, $\widetilde{\mathcal{A}} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5) = (96, 108, 120, 132, 144)$ respectively.

Solution
Taking into account the above information, we acquire the following optimal solutions:

$$T^* = 0.777396, \qquad \mathcal{Q}^* = 70.118, \qquad GTC_4^* = 3007.23.$$



(A) TC_4 vs T (B) GTC_4 vs T

FIGURE 7. Convexities of total cost by employing Example 4

7. SENSITIVITY ANALYSIS

It is crucial to determine the factors influencing the optimal inventory management strategy and create a response plan to mitigate their effects. To this end, we use Example 1 to conduct a sensitivity analysis on the relevant constraints. The constraints will be altered (increased and decreased) within a range of -50% to +50%. One parameter is altered at a time to observe the effects, while the others remain unchanged. The findings are presented in TABLE 2 and FIGURES (8 - 14). Sensitivity analysis helps determine how key variables affect the overall cost functions. Additionally, managerial insights are provided to address various scenarios that could arise during the financial cycle.

TABLE 2. Effects of Varying Constraint Values on Optimal Outcomes

Parameter	Changes (%)	Crisp Model			Fuzzy Model		
		T^*	$\mathcal{Q}^*$	TC_1^*	T^*	$\mathcal{Q}^*$	GTC_1^*
η	-50	1.46224	105.36	1653.91	1.45515	104.90	1676.97
	-25	1.46292	105.41	1653.60	1.45583	104.969	1676.65
	+25	1.46496	105.611	1652.66	1.45789	105.113	1675.69
	+50	1.46546	105.662	1652.34	1.45858	105.164	1675.36
$\mathcal{R}$	-50	1.45887	105.159	1739.47	1.45174	104.656	1763.98
	-25	1.46157	105.36	1696.22	1.45446	104.858	1719.99
	+25	1.46701	105.763	1609.71	1.45996	105.266	1632.02
	+50	1.46975	105.967	1566.46	1.46273	105.472	1588.03
I_C	-50	1.53228	110.615	1649.88	1.5237	110.005	1673.01
	-25	1.49436	107.795	1651.58	1.4866	107.246	1674.67
	+25	1.43982	103.746	1654.12	1.43331	103.289	1677.12
	+50	1.41953	102.242	1655.09	1.41351	101.821	1678.06
I_E	-50	1.53786	111.03	1688.73	1.53007	110.479	1711.94
	-25	1.50155	108.329	1671.06	1.49411	107.803	1694.19
	+25	1.42597	102.72	1634.42	1.41928	102.248	1657.37
	+50	1.38654	99.799	1615.37	1.38025	99.358	1638.24
$\mathcal{P}$	-50	1.42396	124.55	1985.49	1.41828	124.064	2008.53
	-25	1.4386	114.762	1817.31	1.43233	114.272	1840.34
	+25	1.50432	96.906	1492.21	1.49609	96.388	1515.29
	+50	1.56334	88.729	1334.60	1.55344	88.181	1357.76
H_C	-50	1.5928	115.124	1598.46	1.58299	114.424	1621.80
	-25	1.52446	110.033	1626.30	1.51617	109.445	1649.47
	+25	1.41074	101.591	1678.61	1.40466	101.165	1701.53
	+50	1.36271	98.038	1703.32	1.35744	97.677	1726.15
ζ	-50	1.51087	108.195	1632.99	1.50365	107.396	1655.79
	-25	1.48708	106.852	1643.04	1.47992	106.351	1665.96
	+25	1.44242	104.318	1662.79	1.43542	103.821	1685.94
	+50	1.42143	103.121	1672.50	1.4145	102.625	1695.77

Managerial insights.

The resulting managerial insights were obtained by performing a sensitivity analysis on some crucial key parameters. From TABLE 2, it is noticed that, the change in the value of

(i) salvage value (α) influences the optimal strategy of inventory significantly (see FIGURE 8). In particular, a raise in the salvage value (α) implies the raise of total cycle length (T) and order quantities ($\mathcal{Q}$). However, the overall cost functions (TC_1 and GTC_1) decrease. Moreover, a decrease in the salvage value (α) leads to a decrease in total cycle length (T) and order quantities ($\mathcal{Q}$), whereas the overall cost functions (TC_1 and GTC_1) increase. Thus, the inventory managers can make a small increase in the salvage value to minimize the overall cost.

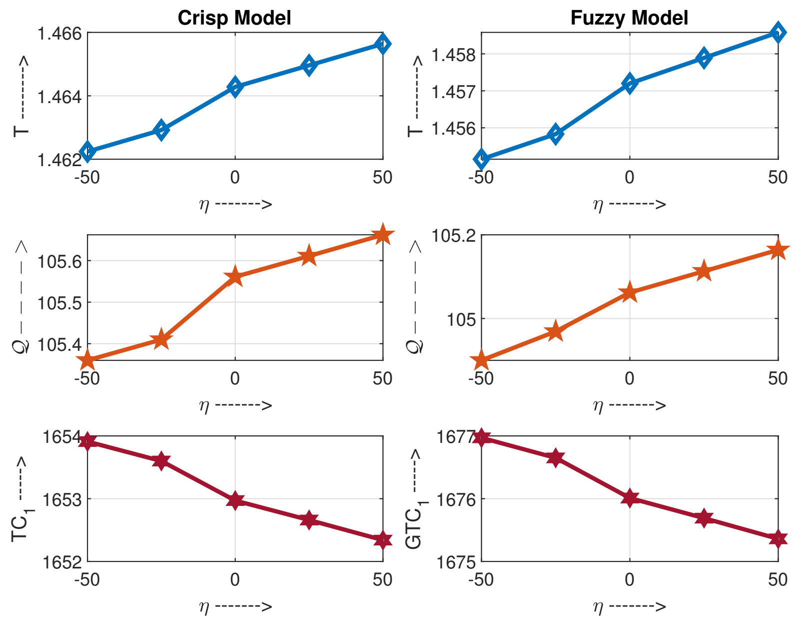


FIGURE 8. Impact of η on optimal results

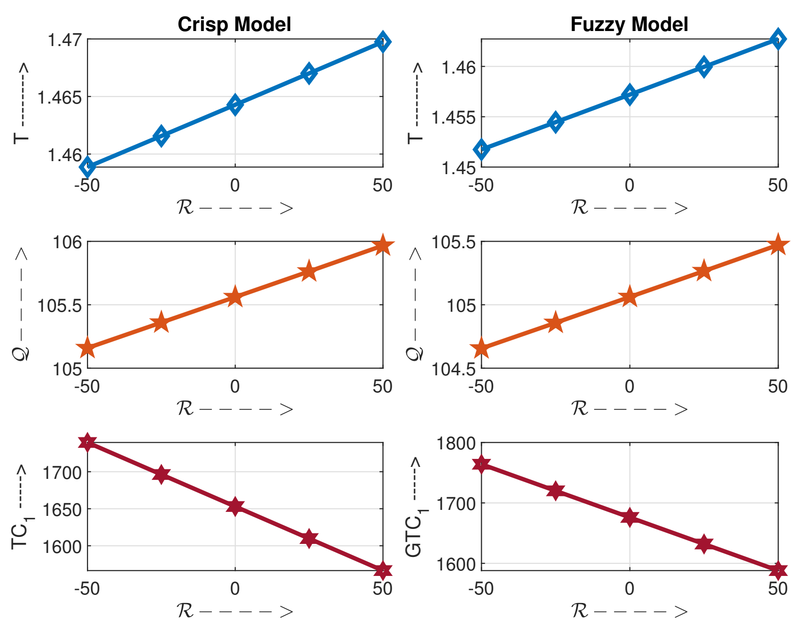


FIGURE 9. Impact of $\mathcal{R}$ on optimal results

(ii) cash discount rate ($\mathcal{R}$) influences the optimal strategy of inventory significantly (see FIGURE 9). In particular, an increase in the cash discount rate ($\mathcal{R}$) implies the increase of total cycle length (T) and order quantities ($\mathcal{Q}$). However, the overall cost functions (TC_1 and GTC_1) decrease. Moreover, a decrease in the cash discount rate ($\mathcal{R}$) leads to a decrease in total cycle length (T) and order quantities ($\mathcal{Q}$), whereas

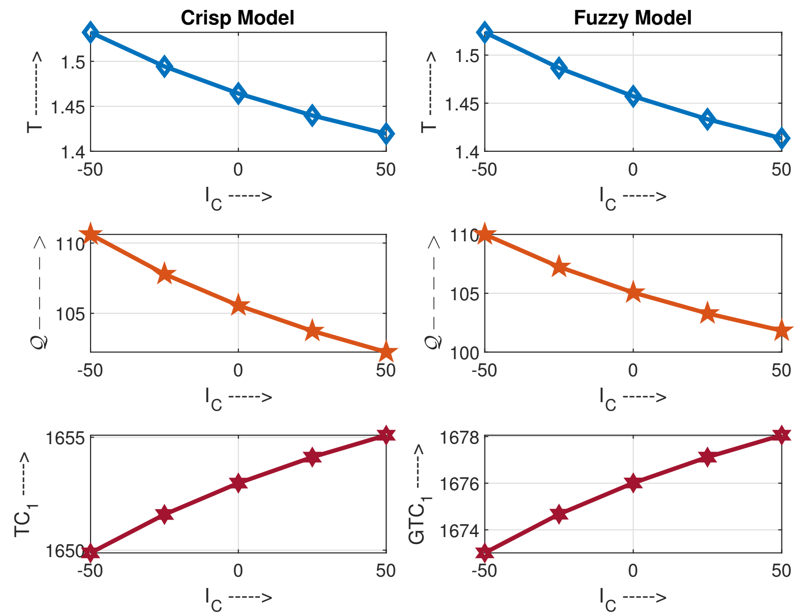


FIGURE 10. Impact of I_C on optimal results

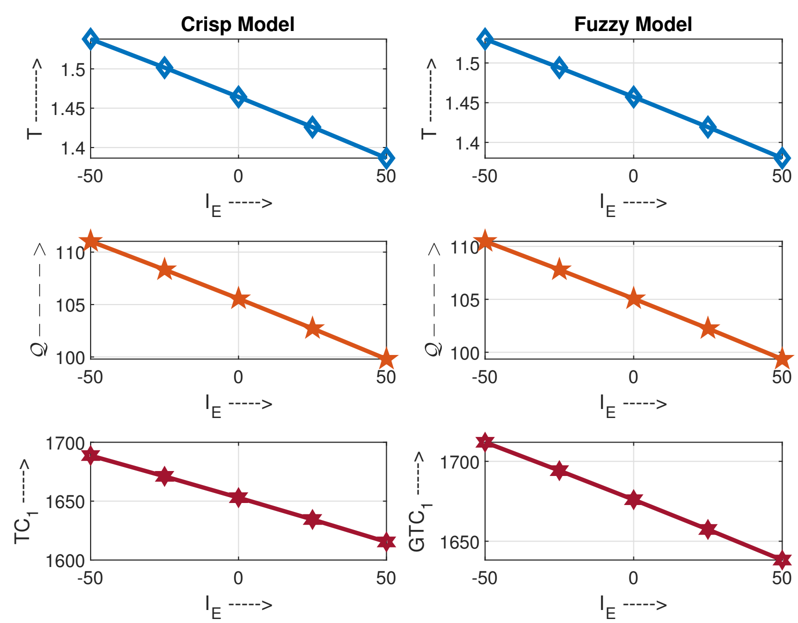


FIGURE 11. Impact of I_E on optimal results

the overall cost functions (TC_1 and GTC_1) increase. Thus, the inventory managers can make a small increase in the cash discount rate to minimize the overall cost.

(iii) interest paid rate (I_C) influences the optimal strategy of inventory significantly (see FIGURE 10). In particular, an increase in the interest paid rate (I_C) implies the decrease of total cycle length (T) and order quantities ($\mathcal{Q}$). However, the overall cost functions (TC_1 and GTC_1) increase. Moreover, a decrease in

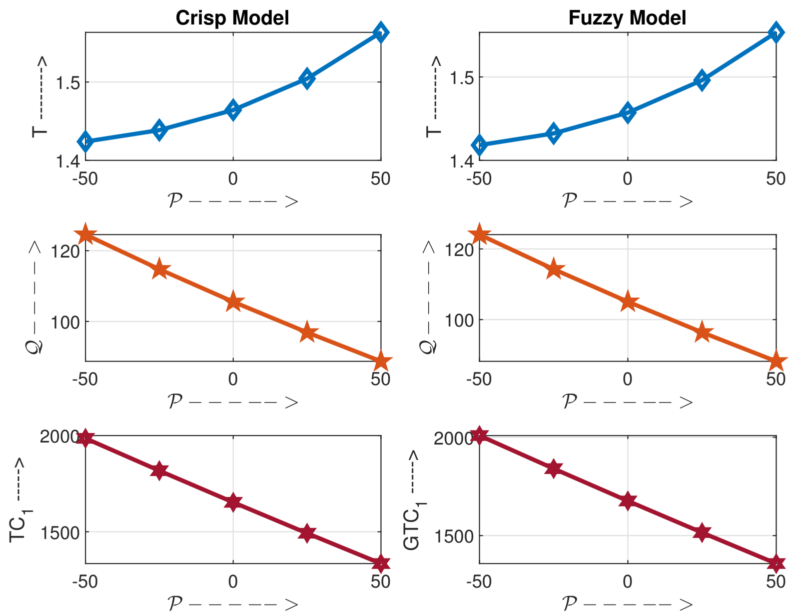


FIGURE 12. Impact of $\mathcal{P}$ on optimal results

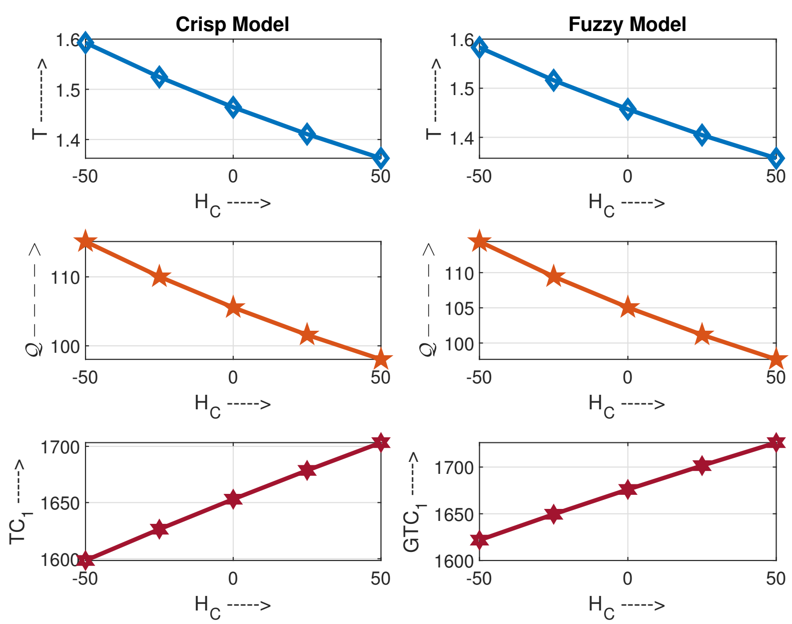


FIGURE 13. Impact of H_C on optimal results

the interest paid rate (I_C) leads to an increase in total cycle length (T) and order quantities ($\mathcal{Q}$), whereas the overall cost functions (TC_1 and GTC_1) decrease. Thus, the inventory managers may try to reduce the interest paid rate to minimize the overall cost.

(iv) interest earned rate (I_E) influences the optimal strategy of inventory significantly (see FIGURE 11). In particular, an increase in the interest earned rate (I_E) implies the decrease of total cycle length (T), order

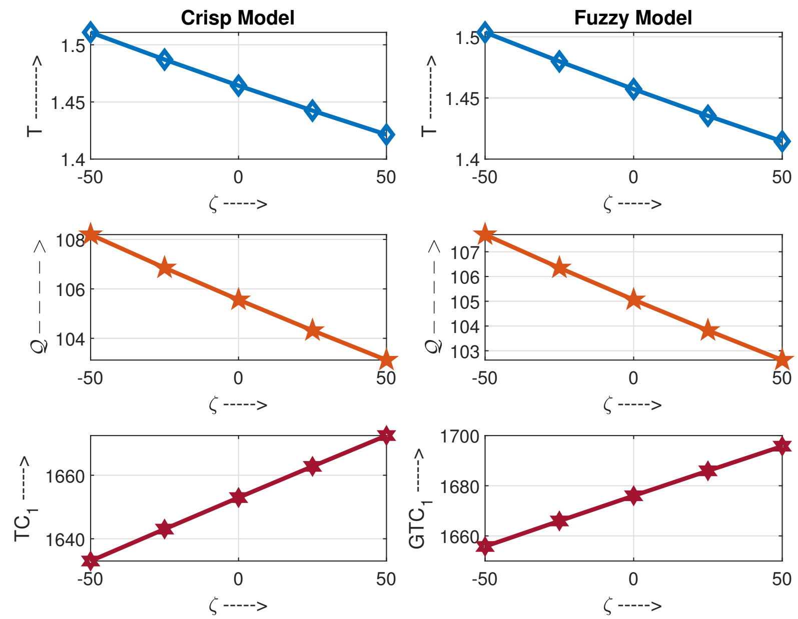


FIGURE 14. Impact of ζ on optimal results

quantities ($\mathcal{Q}$), and overall cost functions (TC_1 and GTC_1). Moreover, a decrease in the interest earned rate (I_E) leads to the increase of total cycle length (T), order quantities ($\mathcal{Q}$), and overall cost functions (TC_1 and GTC_1). Thus, the inventory managers can make a small increase in the interest earned rate to minimize the overall cost.

(v) selling price ($\mathcal{P}$) influences the optimal strategy of inventory significantly (see FIGURE 12). In particular, an increase in the selling price ($\mathcal{P}$) implies the decrease of order quantities ($\mathcal{Q}$) and overall cost functions (TC_1 and GTC_1). However, the total cycle length (T) increases. Moreover, a decrease in the selling price ($\mathcal{P}$) leads to an increase in order quantities ($\mathcal{Q}$) and overall cost functions (TC_1 and GTC_1), whereas the total cycle length (T) decreases. Thus, the inventory managers can make a small increase in the selling price to minimize the overall cost.

(vi) holding cost (H_C) influences the optimal strategy of inventory significantly (see FIGURE 13). In particular, an increase in the holding cost (H_C) implies the decrease of total cycle length (T) and order quantities ($\mathcal{Q}$). However, the overall cost functions (TC_1 and GTC_1) increase. Furthermore, a decrease in the holding cost (H_C) leads to a raise in total cycle length (T) and order quantities ($\mathcal{Q}$), whereas the overall cost functions (TC_1 and GTC_1) decrease. Thus, the inventory managers may try to reduce the holding cost to minimize the overall cost.

(vii) stock-dependent consumption rate (ζ) influences the optimal strategy of inventory significantly (see FIGURE 14). In particular, an increase in the stock-dependent consumption rate (ζ) implies the decrease of total cycle length (T) and order quantities ($\mathcal{Q}$). However, the overall cost functions (TC_1 and GTC_1) increase. Moreover, a decrease in the stock-dependent consumption rate (ζ) leads to an increase in total cycle length (T) and order quantities ($\mathcal{Q}$), whereas the overall cost functions (TC_1 and GTC_1) decrease. Thus, the inventory managers may try to reduce the stock-dependent consumption rate to minimize the overall cost.

8. Conclusion

This study presents an optimized supply-chain inventory framework for deteriorating items, addressing the interplay between stock levels, pricing strategies, and supplier incentives such as price discounts and payment delays. By formulating both deterministic and fuzzy models, we provide a comprehensive approach to managing inventory under varying levels of uncertainty. The deterministic model is ideal for predictable cost environments, ensuring efficient inventory control and cost minimization. Meanwhile, the fuzzy model, incorporating pentagonal fuzzy numbers and GMIR-based defuzzification, offers a robust strategy for handling uncertainties in ordering costs, purchase costs, demand rates, and deterioration rates. Through numerical validation and sensitivity analyses, we demonstrate the practical applicability of our model and provide actionable insights for inventory managers. The findings highlight the benefits of dynamic pricing and credit policies in mitigating stock depletion risks and enhancing profitability. Ultimately, this study equips decision-makers with effective strategies to optimize inventory management, reduce costs, and ensure sustainable operations in a competitive market.

This study highlights several promising avenues for further exploration in the field of inventory models. One potential extension is the development of a two-warehouse inventory model that incorporates two-level trade credit financing, inflation, and various demand patterns, such as those influenced by advertising, power patterns, or selling price. Another valuable direction for future research is the inclusion of factors like shortages, quantity discounts, and other business-related scenarios.

Statements and Declarations

The authors declare that there are no conflicts of interest associated with this study. Additionally, the research did not receive any funding.

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