JOURNAL OF DECISION MAKING AND HEALTHCARE
Volume 2 (2025), No. 2, 136–153
https://doi.org/10.69829/jdmh-025-0202-ta04
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OPTIMAL EOQ INVENTORY MODEL HAVING PRICE SENSITIVE DEMAND WITH WEIBULL DETERIORATION UNDER COMPLETE BACKLOGGED SHORTAGES IN CRISP AND FUZZY ENVIRONMENTS

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Dedicated to Professor Hari Mohan Srivastava on the Occasion of His 85th Birthday

Abstract. This research presents an economic order quantity (EOQ) inventory model for deteriorating products with price-sensitive demand and a Weibull deterioration rate, considering complete backlogged shortages. The model is examined in both crisp and fuzzy environments to address uncertainties in inventory parameters. In the crisp setting, deterministic values are used to determine optimal inventory decisions, whereas the fuzzy approach incorporates triangular fuzzy numbers, with defuzzification performed using the graded mean integration representation (GMIR) method. The framework integrates a backlogging mechanism where the proportion of backlogged demand varies over time, aligning with real-world inventory scenarios. A mathematical optimization technique is applied to establish the optimal replenishment policy that minimizes total inventory costs. Numerical examples validate the model through Mathematica 13.0.1 software, and a sensitivity analysis explores the influence of key parameters on optimal solutions. The managerial insights derived from this study provide valuable strategies for retailers managing perishable goods and operating in price-sensitive markets.

Keywords. EOQ, Price-sensitive demand, Backlogging, Weibull deterioration, Graded mean integration representation method, Triangular fuzzy numbers.

1. Introduction

Efficient inventory management is crucial for businesses handling perishable goods, especially when demand is influenced by price and product deterioration is inevitable. In such cases, inventory models must consider key factors such as product decay, demand responsiveness, and shortages to develop cost-effective replenishment strategies. The Economic Order Quantity (EOQ) model is a widely used framework for optimizing order quantities to minimize total inventory costs. However, conventional EOQ models typically assume a constant demand rate and neglect product deterioration, limiting their applicability to real-world scenarios where demand fluctuates with price and products degrade over time.

In recent times, various approaches have been employed to address real-world challenges, particularly in inventory control. A key concern for management is determining when and how much to order or produce to minimize the total cost associated with the inventory system. This issue becomes especially critical when inventory is subject to decay or deterioration, which includes changes, damage, spoilage, obsolescence, and a reduction in usefulness from its original value. It is well known that items such as vegetables, medicine, gasoline, blood, and radioactive chemicals deteriorate over time

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Accepted: April 06, 2025.

when stored. Therefore, when formulating an optimal inventory policy for such products, potential losses due to deterioration must be carefully considered.

In most cases, precise data from real-world scenarios are insufficient to fully support a mathematical model. Human judgments, including preferences, are often imprecise and cannot always be quantified with exact numerical values. Fuzzy sets offer a versatile approach for representing gradual transitions between membership and non-membership functions, providing a more detailed and accurate depiction of uncertainty. This approach effectively captures vague concepts frequently encountered in decision-making while offering a crucial approximation of instabilities. The fundamental concept of a fuzzy set, which extends the classical or crisp set, is both simple and elegant. L. A. Zadeh [44] introduced the notion of fuzzy sets as an extension of traditional set theory. Conventional inventory models typically incorporate factors such as holding costs, deterioration, demand, and other constraints based on available historical data. However, due to ambiguity and incomplete information, these decisions may not always be precise when applied to real-world scenarios. To address these uncertainties in inventory constraints, many researchers have incorporated fuzziness into their models.

Classical inventory models typically assume a constant demand rate. However, in real-world scenarios, the demand for physical goods is influenced by various factors such as time, stock availability, and price. Researchers have explored diverse demand patterns, including those driven by time, advertising costs, selling price, and trade credit financing. Burwell *et al.* [4] examined an inventory system where demand depends on price, allowing for fluctuations either increasing, decreasing, or remaining constant throughout the cycle. The selling price plays a critical role in such systems. Several studies have further investigated price-dependent demand in inventory models. For instance, Mondal *et al.* [21], Routray *et al.* [34], You *et al.* [43], and Barik *et al.* [1] analyzed inventory systems for ameliorating items with price-sensitive demand. Additional contributions on this topic include studies by Barik *et al.* [2, 3], and Indrajitsingha *et al.* [6, 7, 8]. Beyond price-dependent demand, researchers have also investigated inventory models with time-dependent demand variations. Jalan *et al.* [10] and Sarkar *et al.* [38] studied inventory systems where demand increases over time. This line of research was further explored by Sarkar *et al.* [39], who provided additional insights into time-dependent demand patterns. Additionally, for further studies on several inventory restraints, one may look into the works of Jani *et al.* [11], Mishra *et al.* [18, 19, 20], Padhy *et al.* [27], Paikray *et al.* [28, 29, 30], Patra *et al.* [31, 32, 33].

In mathematical inventory models, costs such as holding, ordering, and deterioration are typically assumed to be fixed. However, in reality, these costs fluctuate due to factors such as changes in petrochemical prices and taxes. As a result, most inventory costs are not precisely defined. Recent research has explored inventory problems under various demand conditions and constraints. For example, Jaggi *et al.* [9] examined inventory models with exponentially decreasing demand, while Kumar *et al.* [13] investigated models with fuzzy demand. Traditionally, researchers have relied on historical data to model inventory constraints using fixed parameters. However, this approach is often inadequate, as business environments and technological advancements are constantly evolving. Consequently, inventory parameters fluctuate over time, making conventional inventory models less effective in addressing modern challenges. To overcome these limitations, researchers have increasingly turned to fuzzy set theory and its applications in inventory management. Over the past decade, numerous studies have explored inventory models within a fuzzy framework, including those by Kumar and Rajput [17], Routray *et al.* [35], Saha [36], Sangal *et al.* [37], Sen *et al.* [40], and Sharmila and Uthayakumar [42]. Readers interested in further insights may refer to the works of Kumar *et al.* [12, 14, 15, 16], Nayak *et al.* [22, 23, 24, 25, 26], and Shaikh *et al.* [41].

Many existing inventory models assume constant demand, often overlooking the dynamic nature of demand, particularly its sensitivity to price fluctuations. In real-world scenarios, demand is influenced by various factors, including market conditions, customer behavior, and competitive dynamics. Previous studies typically simplify demand models by assuming fixed or linear demand patterns, which fail to capture the complexity of real-world inventory systems. Additionally, most traditional inventory models treat key parameters, such as deterioration rates, holding costs, and order costs, as deterministic values. However, in practice, these parameters fluctuate due to market uncertainties, environmental factors, and operational risks. The lack of consideration for such uncertainties significantly limits the applicability of these models in real-world settings. Although some studies have attempted to incorporate uncertainty, many rely on oversimplified or limited methods, such as assuming single-point estimates for imprecise parameters, which fail to fully represent the variability inherent in inventory systems. Furthermore, while backlogging is a crucial aspect of inventory management, many studies do not adequately account for the time-varying nature of backlogged demand. Often, the assumption of a constant backlogging rate or complete backlogging is made, which does not reflect the reality of most inventory systems. In practice, the rate of demand backlogging is influenced by factors such as customer satisfaction and external conditions, which can change over time. This limitation diminishes the relevance of previous models, particularly for industries dealing with perishable goods or high-demand fluctuations.

To address these challenges, this study proposes an EOQ inventory model for deteriorating products with price-sensitive demand and a Weibull deterioration rate, while accounting for complete backlogged shortages. The Weibull deterioration function is chosen for its adaptability in modeling various decay patterns observed in perishable goods. Furthermore, the model integrates a backlogging mechanism where the proportion of backlogged demand changes over time, providing a more realistic depiction of inventory shortages. Key inventory parameters, including deterioration cost, purchase cost, ordering cost, and holding costs, are subject to uncertainty due to market fluctuations and environmental influences. To accommodate these uncertainties, this research analyzes the model in both crisp and fuzzy environments. In the crisp framework, parameters are considered deterministic, allowing for precise cost optimization. In contrast, the fuzzy approach incorporates triangular fuzzy numbers to represent imprecise parameters, with defuzzification carried out using the graded mean integration representation method. This approach enables decision-makers to develop optimal strategies under uncertain conditions. A mathematical optimization technique is employed to determine the optimal replenishment policy that minimizes total inventory costs. The model is validated through numerical illustrations, and a sensitivity analysis is conducted to examine the influence of key parameters on the optimal solutions.

The remainder of this paper is structured as follows: Section 2 presents fundamental definitions related to fuzzy set theory. Following this, Section 3 outlines the assumptions and nomenclature necessary for framing the problem. Section 4 focuses on the mathematical framework of the model within a crisp environment. The computational solution algorithm for the problem is detailed in Section 5. Section 6 extends the discussion to the model's formulation in fuzzy environments. Section 7 demonstrates the practical applicability of the study through numerical examples. Section 8 provides sensitivity analysis and valuable managerial insights. Finally, Section 9 concludes the study and highlights potential directions for future research.

2. Definitions and Preliminaries

In this section, we recall the following definitions that are necessary to study the proposed model.

Definition 2.1. [45]

Let X be a space of points and $\mu : X \to [0, 1]$ be such that for every $x \in X$, $\mu(x)$ is a real number in the interval $[0, 1]$. We define a fuzzy set $\widetilde{A}$ in X as the ordered pair $\widetilde{A} = \{ (x, \mu_{\widetilde{A}}(x)) : x \in X \}$, where x is called a generic element and $\mu_{\widetilde{A}}(x)$ a membership function.

Definition 2.2. [45]

A given fuzzy set $\widetilde{A} = \{x, \mu_{\widetilde{A}}(x)\} \subseteq X$ is termed a convex fuzzy set if all the $\alpha - cut$ sets are convex for every $x \in X$. That is, for every pair of elements $x_1, x_2 \in X$,

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}, \forall\, \lambda \in [0,1].$$

Definition 2.3. [45]
Let $a, b \in R$ such that $a < b$. Then, the fuzzy set $[a_\alpha, b_\alpha]$ is called a fuzzy interval for $0 \leq \alpha \leq 1$ if its membership function is

$$\mu_{[a_\alpha, b_\alpha]} = \begin{cases} \alpha, & a \leq x \leq b \\ 0, & otherwise \end{cases}.$$

Definition 2.4. [45]
Let $a, b, c \in R$ such that $a < b < c$. Then a fuzzy number $\widetilde{A} = (a, b, c)$ is called a triangular fuzzy number if its membership function is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & otherwise \end{cases}.$$

In particular, when $a = b = c$, $(c, c, c) = c$, this is called a fuzzy point. The family of all triangular fuzzy numbers on R is usually denoted as

$$F_N = \{(a, b, c) : a < b < c, \forall\, a, b, c \in R\}.$$

The $\alpha-$ cut of $\widetilde{A} = (a, b, c) \in F_N, 0 \leq \alpha \leq 1$, usually denoted by $A(\alpha)$, is defined as $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$, where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = c - (c-b)\alpha$ are the left and right endpoints of $A(\alpha)$ respectively.

Definition 2.5. [45]
The graded mean integration representation (GMIR) of $\widetilde{A}$, given a triangular fuzzy number $\widetilde{A} = (a, b, c)$, is defined as

$$P\left(\widetilde{A}\right) = \frac{\int_0^{W_A} h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right) dh}{\int_0^{W_A} h dh}$$

with $0 < h < W_A$ and $0 < W_A \leq 1$.
That is,

$$P\left(\widetilde{A}\right) = \frac{1}{6}(a + 4b + c).$$

3. Assumptions and Nomenclature

The proposed inventory problem is mathematically formulated based on the following set of assumptions and defined nomenclature.

3.1. Assumptions.

(i) The inventory system involves the exchange of uniform items.
(ii) For the crisp model, the related costs are deterministic, whereas for the fuzzy model, they are imprecise.
(iii) The demand rate is price dependent and is of the form $D(p) = ap^{-b}$, $a, b > 0$.
(iv) The lead-time is zero and the time horizon is infinite.
(v) The replenishment rate is instantaneous.
(vi) Shortages are allowed to occur which are complete backordered.

(vii) The rate of deterioration at any time $t > 0$ follows the two parameter Weibull distribution as $\theta = \alpha\beta t^{\beta-1}$, where $\alpha(0 < \alpha < 1)$ is the scale parameter and $\beta > 0$ is the shape parameter.
(viii) In the fuzzy environment, imprecise costs are represented using triangular fuzzy numbers.

3.2. **Nomenclature.**

(i) D : demand rate.
(ii) p : selling price.
(iii) α : scale parameter.
(iv) β : shape parameter.
(v) M_1 : maximum inventory per cycle.
(vi) M : maximum shortage level.
(vii) $\mathbb{W}$: order quantities.
(viii) OC : total ordering cost per cycle.
(ix) A : ordering cost per unit time.
(x) DC : total deterioration cost per cycle.
(xi) Dp : deteriorating cost per unit time.
(xii) HC : total holding cost per cycle.
(xiii) Hp : holding cost per unit time.
(xiv) PC : total purchase cost per cycle.
(xv) Cp : purchase cost per unit time.
(xvi) SC : total Shortage cost per cycle.
(xvii) Sp : shortage cost per unit time.
(xviii) t_1 : time at which the inventory level reaches zero.
(xix) T : total cycle length.
(xx) $TC(t_1)$: total inventory cost in crisp environments.
(xxi) $\widetilde{A}$: ordering cost per unit time in fuzzy environments.
(xxii) $\widetilde{Hp}$: holding cost per unit time in fuzzy environments.
(xxiii) $\widetilde{p}$: selling price in fuzzy environments.
(xxiv) $\widetilde{Cp}$: purchase cost per unit time in fuzzy environments.
(xxv) $\widetilde{Sp}$: shortage cost per unit time in fuzzy environments.
(xxvi) $\widetilde{TC}(t_1)$: total inventory cost in in fuzzy environments.
(xxvii) $GTC(t_1)$: defuzzified total cost.

4. Formulation of Mathematical Model

The objective of the framework is to identify the optimal order quantity that results in the lowest total cost. The inventory begins at $t = 0$ with $\mathbb{W}$ items, out of which M_1 items are used for the inventory cycle $[0, t_1]$, and the remaining M items will be used to meet the cycle's backorder in the $[t_1, T]$ interval. The stock diminishes over time owing to the incorporated effects of deterioration and demand in $[0, t_1]$, and the inventory will reach zero at time t_1. So, the stock deficiency occurs in the interval $[t_1, T]$. The subsequent replenishment only replaces the backlogged items. Hence, the inventory level at any time t is illustrated in FIGURE 1 and is expressed in the following mathematical expression.

Let $Q_1(t)$ be the accessible inventory in the interval $[0, t_1]$. The inventory reduces owing to deterioration and demand in the interval $[0, t_1]$. Therefore, the inventory level at any time t within the interval $[0, t_1]$ is characterized by the following differential equation

$$\frac{dQ_1(t)}{dt} + \alpha\beta t^{\beta-1}Q_1(t) = -ap^{-b},\ 0 \leq t \leq t_1 \tag{4.1}$$

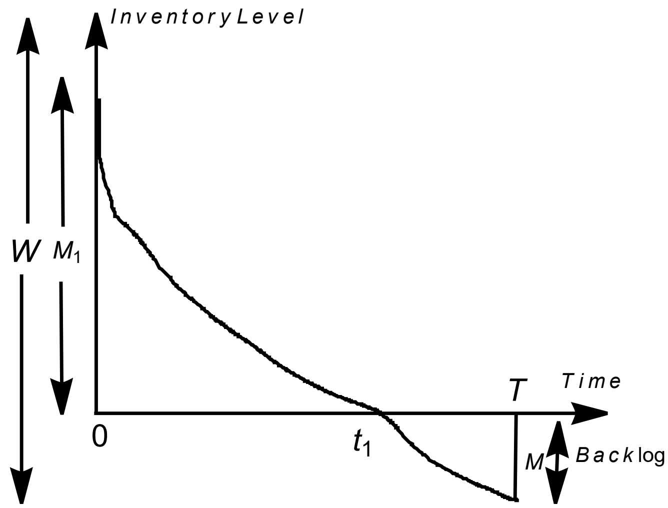

FIGURE 1. Inventory level at any time t

with the boundary condition $Q_1(t_1) = 0$.

The preceding differential equation's solution is provided by,

$$Q_1(t) = ap^{-b}\left\{(t_1 - t)\left(1 - \alpha t^{\beta}\right) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right)\right\}. \tag{4.2}$$

Let $Q_2(t)$ be the on-hand inventory in the interval $[t_1, T]$. In the interval $[t_1, T]$, shortages occur and are completely backlogged. Therefore, inventory has a negative stock balance at this time. Thus, the inventory level at any instance t within the interval $[t_1, T]$ is represented by the differential equation

$$\frac{dQ_2(t)}{dt} = -ap^{-b},\ t_1 \le t \le T \tag{4.3}$$

with the boundary constrain $Q_2(t_1) = 0$.

The solution to the foregoing differential equation is presented by,

$$Q_2(t) = ap^{-b}\left(t_1 - t\right). \tag{4.4}$$

The maximum inventory level during $[0, t_1]$ is

$$M_1 = Q_1(t=0) = ap^{-b}\left\{t_1 + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right\}. \tag{4.5}$$

The maximum backlogged level per cycle is

$$M = -Q_2(t=T) = ap^{-b}\left(T - t_1\right). \tag{4.6}$$

Thus, the order quantity over the replenishment cycle is $\mathbb{W} = M_1 + M$, that is,

$$\mathbb{W} = ap^{-b}\left(T + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right). \tag{4.7}$$

Total number of deteriorating units during the cycle $(0, t_1)$ is given by

$$D_\theta = \int_0^{t1} \theta\, Q_1(t)dt = ap^{-b}\frac{\alpha}{\beta+1}\, t_1^{\beta+1}. \tag{4.8}$$

The overall cost for each replenishment cycle is calculated using the cost components shown below.

Ordering Cost.

$$OC = A. \tag{4.9}$$

Deterioration Cost.

$$DC = Dp \times D_\theta = Dp\left(ap^{-b}\frac{\alpha}{\beta+1}\,t_1^{\beta+1}\right). \tag{4.10}$$

Holding Cost.

$$HC = Hp\int_0^{t_1} Q_1(t)dt = Hp\left(ap^{-b}\left\{\frac{t_1^2}{2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\right\}\right). \tag{4.11}$$

Purchase Cost.

$$PC = Cp \times \mathbb{W} = Cp\left(ap^{-b}\left\{T+\frac{\alpha}{\beta+1}t_1^{\beta+1}\right\}\right). \tag{4.12}$$

Shortage Cost.

$$SC = -Sp\int_{t_1}^{T} Q_2(t)dt = Sp\left(ap^{-b}\left\{\frac{T^2}{2}-T\,t_1+\frac{t_1^2}{2}\right\}\right). \tag{4.13}$$

Now, the total cost per unit time becomes,

$$\begin{aligned} TC(t_1) &= \frac{1}{T}\Big\{OC+DC+HC+PC+SC\Big\}\\ &= \frac{1}{T}\Big\{A+Dp\left(ap^{-b}\frac{\alpha}{\beta+1}t_1^{\beta+1}\right)+Hp\left(ap^{-b}\left\{\frac{t_1^2}{2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\right\}\right)\\ &\quad +Cp\left(ap^{-b}\left\{T+\frac{\alpha}{\beta+1}t_1^{\beta+1}\right\}\right)+Sp\left(ap^{-b}\left\{\frac{T^2}{2}-T\,t_1+\frac{t_1^2}{2}\right\}\right)\Big\}. \end{aligned} \tag{4.14}$$

5. Computational Algorithm

The conventional optimal approach has been adopted to solve the problem. The basic objective is to minimize the overall cost function TC. The steps to verify the optimal decision parameters leading to the overall minimum cost are as follows:

Step 1 Initialize the inventory parameters; $a, b, A, \alpha, \beta, p, T, Hp, Sp, Cp$, and Dp.
Step 2 Find $TC(t_1)$.
Step 3 Determine $\frac{\partial TC(t_1)}{\partial t_1}$.
Step 4 Resolve the equation $\frac{\partial TC(t_1)}{\partial t_1} = 0$ for t_1.
Step 5 Select the solution from Step 4.
Step 6 Determine $\frac{\partial^2 TC(t_1)}{\partial t_1^2}$.
Step 7 Check if $\frac{\partial^2 TC(t_1)}{\partial t_1^2} > 0$, then the solution is optimal (minimum).
Step 8 Otherwise proceed to Step 5.

6. Fuzzy Model

In practical situations, inventory parameters are frequently uncertain because of various influencing factors. For instance, it is difficult to define every parameter accurately because of fuzziness. To account for the imprecision in various parameters, we suggest a model that operates within fuzzy contexts. Specifically, for the fuzzy model, we introduce fuzzy parameters $\widetilde{A}$, $\widetilde{Hp}$, $\widetilde{p}$, $\widetilde{Cp}$, and $\widetilde{Sp}$, which correspond to the ordering cost A, holding cost Hp, Selling price p, Purchase cost Cp, and shortage cost Sp in our crisp framework.

The fuzzy total cost is given by

$$\widetilde{TC}(t_1) = \frac{1}{T}\Bigg\{\widetilde{A} + Dp\, a\widetilde{p}^{-b}\frac{\alpha}{\beta+1}\, t_1^{\beta+1} + \widetilde{Hp}\, a\widetilde{p}^{-b}\Bigg\{\frac{t_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\Bigg\} + \widetilde{Cp}\, a\widetilde{p}^{-b}\left(T + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right) + \widetilde{Sp}\, a\widetilde{p}^{-b}\left(\frac{T^2}{2} - T\,t_1 + \frac{t_1^2}{2}\right)\Bigg\}. \tag{6.1}$$

6.1. Defuzzification.

By using triangular fuzzy numbers, the fuzzy parameters are outlined as $\widetilde{A} = (A_1, A_2, A_3)$, $\widetilde{Hp} = (Hp_1, Hp_2, Hp_3)$, $\widetilde{p} = (p_1, p_2, p_3)$, $\widetilde{Cp} = (Cp_1, Cp_2, Cp_3)$ and $\widetilde{Sp} = (Sp_1, Sp_2, Sp_3)$. Thus, following Chen and Hsieh [5] the total fuzzy cost function is defuzzified under the graded mean integration representation method, leading to the total defuzzified cost as

$$GTC(t_1) = \frac{1}{4}\Bigg\{\widetilde{TC}_1 + 4\times\widetilde{TC}_2 + \widetilde{TC}_3\Bigg\}. \tag{6.2}$$

Here, $\widetilde{TC}_i$ is obtained from the above equation (6.1) just by replacing the imprecise parameters in $\widetilde{TC}$ with the corresponding i^{th} triangular fuzzy number for $i = 1, 2, 3$.
That is,

$$\begin{aligned} GTC(t_1) = \frac{1}{4}\Bigg\{&\frac{1}{T}\Bigg\{\widetilde{A_1} + Dp\, a\widetilde{p_1}^{-b}\frac{\alpha}{\beta+1}\, t_1^{\beta+1} + \widetilde{Hp_1}\, a\widetilde{p_1}^{-b}\Bigg\{\frac{t_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\Bigg\} \\ &+ \widetilde{Cp_1}\, a\widetilde{p_1}^{-b}\left(T + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right) + \widetilde{Sp_1}\, a\widetilde{p_1}^{-b}\left(\frac{T^2}{2} - T\,t_1 + \frac{t_1^2}{2}\right)\Bigg\} \\ &+ 4\times\Bigg\{\frac{1}{T}\Bigg\{\widetilde{A_2} + Dp\, a\widetilde{p_2}^{-b}\frac{\alpha}{\beta+1}\, t_1^{\beta+1} + \widetilde{Hp_2}\, a\widetilde{p_2}^{-b}\Bigg\{\frac{t_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\Bigg\} \\ &+ \widetilde{Cp_2}\, a\widetilde{p_2}^{-b}\left(T + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right) + \widetilde{Sp_2}\, a\widetilde{p_2}^{-b}\left(\frac{T^2}{2} - T\,t_1 + \frac{t_1^2}{2}\right)\Bigg\}\Bigg\} \\ &+ \frac{1}{T}\Bigg\{\widetilde{A_3} + Dp\, a\widetilde{p_3}^{-b}\frac{\alpha}{\beta+1}\, t_1^{\beta+1} + \widetilde{Hp_3}\, a\widetilde{p_3}^{-b}\Bigg\{\frac{t_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\Bigg\} \\ &+ \widetilde{Cp_3}\, a\widetilde{p_3}^{-b}\left(T + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right) + \widetilde{Sp_3}\, a\widetilde{p_3}^{-b}\left(\frac{T^2}{2} - T\,t_1 + \frac{t_1^2}{2}\right)\Bigg\}\Bigg\}. \end{aligned} \tag{6.3}$$

We can find the optimal solution for the fuzzy model by following a process similar to that of the solution process of crisp model.

7. Numerical Illustrations

We numerically investigated the model in both crisp and fuzzy contexts under our proposed method of solution. We utilize Mathematica 13.0.1 software to get the optimal solution and subsequently demonstrate the convexities of the total cost functions with respect to positive inventory time and order quantity as depicted in FIGURES (2 - 9). The data used in this model are from some peer-review journals and are of hypothetical in nature.

Example 1

(a) ***Crisp Model***

The values of different parameters involved in inventory are $a = 10$, $b = 1$,$A = 200$, $\alpha = 0.005$, $\beta = 0.4$, $p = 6$,$T = 10$, $Hp = 0.5$, $Sp = 0.2$, $Cp = 2$, $Dp = 2$.

Solution

The above-mentioned data yields the following feasible solutions:

$$t_1 = 3.9250 \qquad \mathbb{W} = 16.7070 \qquad TC = 24.3505.$$

(b) ***Fuzzy Model***

The values of different parameters involved in inventory are $a = 10$, $b = 1$, $\alpha = 0.005$, $\beta = 0.4$, $Dp = 2$,$T = 10$ $A_1 = 160$, $A_2 = 200$, $A_3 = 240$, $p_1 = 4.8$, $p_2 = 6$, $p_3 = 7.2$, $Hp_1 = 0.4$, $Hp_2 = 5$, $Hp_3 = 0.6$,$Cp_1 = 1.6$, $Cp_2 = 2$, $Cp_3 = 2.4$, $Sp_1 = 0.16$, $Sp_2 = 2$, $Sp_3 = 0.24$.

Solution

The above-mentioned data yields the following feasible solutions:

$$t_1 = 3.9248 \qquad \mathbb{W} = 16.7633 \qquad GTC = 24.3618.$$

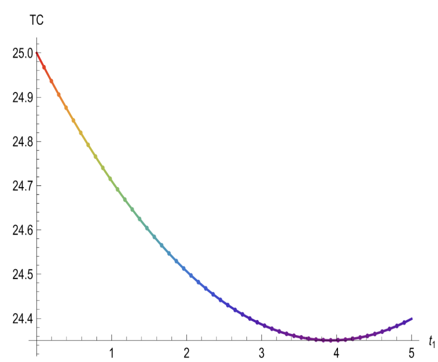

FIGURE 2. Convexity of TC with respect to t_1

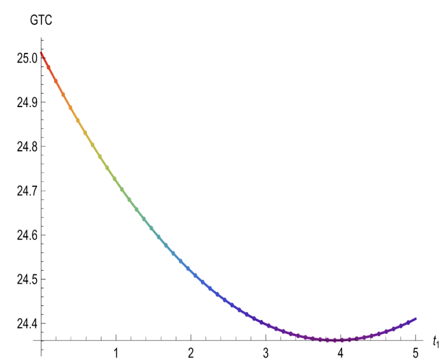

FIGURE 3. Convexity of GTC with respect to t_1

Example 2

(a) ***Crisp Model***

The values of different parameters involved in inventory are $a = 15$, $b = 5$,$A = 200$, $\alpha = 0.02$, $\beta = 0.1$, $p = 2$, $T = 5$, $Dp = 4$, $Hp = 0.2$, $Cp = 1.5$, $Sp = 2$.

Solution

The above-mentioned data yields the following feasible solutions:

$$t_1 = 4.06723 \qquad \mathbb{W} = 2.38363 \qquad TC = 21.1601.$$

(b) ***Fuzzy Model***

The values of different parameters involved in inventory are $a = 15$, $b = 5$, $\alpha = 0.02$, $\beta = 0.1$, $Dp = 4$,$T = 5$, $A_1 = 90$, $A_2 = 100$, $A_3 = 110$, $p_1 = 1.8$, $p_2 = 2$, $p_3 = 2.2$, $Hp_1 = 0.18$, $Hp_2 = 0.2$, $Hp_3 = 0.22$, $Cp_1 = 1.35$, $Cp_2 = 1.5$, $Cp_3 = 1.65$, $Sp_1 = 1.8$, $Sp_2 = 2$, $Sp_3 = 2.2$.

Solution

The above-mentioned data yields the following feasible solutions:

$$t_1 = 4.09034 \qquad \mathbb{W} = 2.50881 \qquad GTC = 21.2020.$$

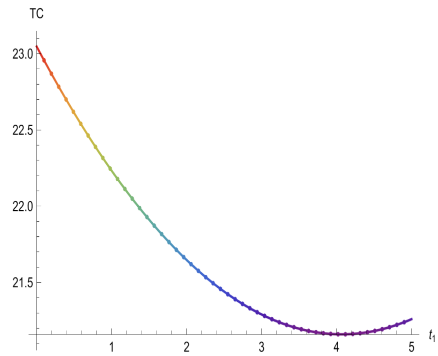

FIGURE 4. Convexity of TC with respect to t_1

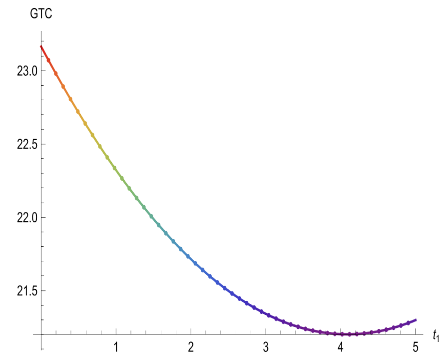

FIGURE 5. Convexity of GTC with respect to t_1

Example 3

(a) ***Crisp Model***

The values of different parameters involved in inventory are $a = 8$, $b = 3$, $A = 120$, $\alpha = 0.2$, $\beta = 0.1$, $p = 8$, $T = 12$, $Dp = 7$, $Hp = 5$, $Cp = 1.5$, $Sp = 4$.

Solution

The above-mentioned data yields the following feasible solutions:

$$t_1 = 0.14099 \qquad \mathbb{W} = 0.18782 \qquad TC = 10.4176.$$

(b) ***Fuzzy Model***

The values of different parameters involved in inventory are $a = 8$, $b = 3$, $\alpha = 0.2$, $\beta = 0.1$, $Dp = 7$,$T = 12$, $A_1 = 108$, $A_2 = 120$, $A_3 = 132$, $p_1 = 7.2$, $p_2 = 8$, $p_3 = 8.8$, $Hp_1 = 4.5$, $Hp_2 = 5$, $Hp_3 = 5.5$, $Cp_1 = 2.7$, $Cp_2 = 3$, $Cp_3 = 3.3$, $Sp_1 = 3.6$, $Sp_2 = 4$, $Sp_3 = 4.4$.

Solution

The above-mentioned data yields the following feasible solutions:

$$t_1 = 0.14235 \qquad \mathbb{W} = 0.19168 \qquad GTC = 10.4223.$$

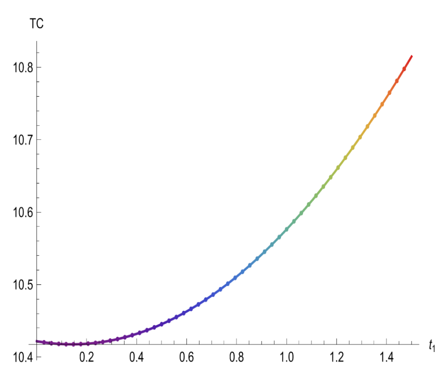

FIGURE 6. Convexity of TC with respect to t_1

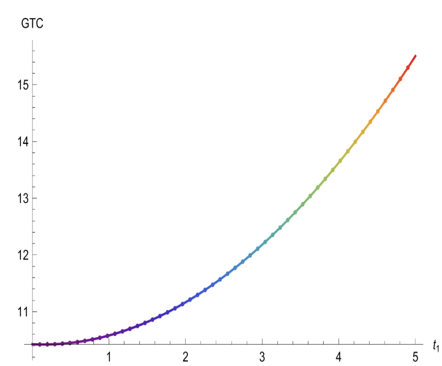

FIGURE 7. Convexity of GTC with respect to t_1

Example 4

(a) *Crisp Model*
The values of different parameters involved in inventory are $a = 15$, $b = 4$, $A = 60$, $\alpha = 0.02$, $\beta = 0.2$, $p = 1$, $T = 15$, $Dp = 4$, $Hp = 3$, $Cp = 7$, $Sp = 3$.

Solution
The above-mentioned data yields the following feasible solutions:

$$t_1 = 13.9411 \qquad \mathbb{W} = 230.903 \qquad TC = 134.546.$$

(b) *Fuzzy Model*
The values of different parameters involved in inventory are $a = 15$, $b = 4$, $\alpha = 0.02$, $\beta = 0.2$, $Dp = 4$,$T = 15$, $A_1 = 54$, $A_2 = 60$, $A_3 = 66$, $p_1 = 0.9$, $p_2 = 1$, $p_3 = 1.1$, $Hp_1 = 2.7$, $Hp_2 = 3$, $Hp_3 = 3.3$, $Cp_1 = 6.3$, $Cp_2 = 7$, $Cp_3 = 7.7$, $Sp_1 = 2.7$, $Sp_2 = 3$, $Sp_3 = 3.3$.

Solution
The above-mentioned data yields the following feasible solutions:

$$t_1 = 13.9571 \qquad \mathbb{W} = 238.885 \qquad GTC = 138.355.$$

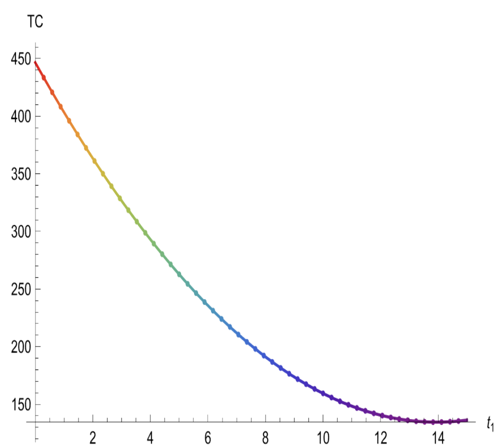

FIGURE 8. Convexity of TC with respect to t_1

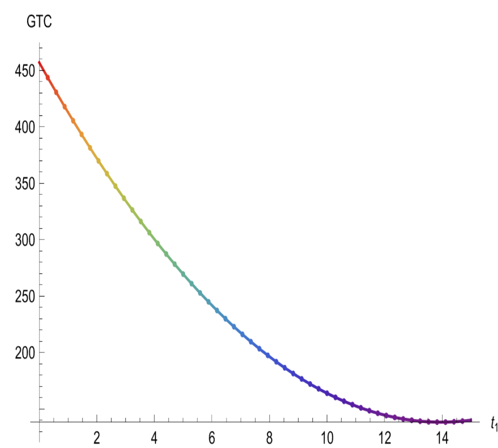

FIGURE 9. Convexity of GTC with respect to t_1

8. SENSITIVITY ANALYSIS

It is highly desirable to pinpoint the variables that govern the ideal strategy for inventory management as well as the response strategy for their influences. In light of this, we will arbitrarily take Example 4 into consideration when performing the sensitivity analysis on related parameters. The parameters are altered (increased and decreased) by -20% to $+20\%$ to do the analysis. One parameter at a time is altered to get the desired outcomes, while the other parameters are left unchanged. We then provide the findings in the TABLE 1 and FIGURES (10 - 14). However, the managerial insights required to handle a variety of circumstances that could occur during the company cycle are also offered.

TABLE 1. Impact of Change in Parameter Values on the Optimal Results

Parameter	Changes(%)	Crisp Model			Fuzzy Model		
		t_1	$\mathbb{W}$	TC	t_1	$\mathbb{W}$	GTC
a	-20	13.7259	184.635	112.284	13.7457	191.019	115.341
	-10	13.8446	207.769	123.432	13.8623	214.951	126.864
	+10	14.0211	254.038	145.636	14.0356	262.818	149.822
	+20	14.0884	277.174	156.707	14.1018	286.753	161.270
Dp	-20	13.9496	230.908	134.231	13.9657	238.889	138.029
	-10	13.9454	230.905	134.389	13.9614	238.887	138.192
	+10	13.9369	230.901	134.704	13.9528	238.882	138.518
	+20	13.9327	230.899	134.861	13.9486	238.880	138.681
α	-20	13.9654	229.733	133.659	13.9817	237.672	137.437
	-10	13.9532	230.319	134.103	13.9694	238.279	137.897
	+10	13.9290	231.487	134.989	13.9449	239.489	138.813
	+20	13.9169	232.069	135.431	13.9324	240.092	139.271
β	-20	13.9540	230.502	134.223	13.9702	238.468	138.021
	-10	13.9477	230.698	134.381	13.9638	238.672	138.672
	+10	13.9342	231.117	134.719	13.9501	239.106	138.533
	+20	13.9268	231.340	134.899	13.9426	239.337	138.720
T	-20	11.1349	184.508	131.110	11.1474	190.885	134.906
	-10	12.5379	207.698	132.777	12.5522	214.877	136.580
	+10	15.3445	254.123	136.390	15.3623	262.907	140.204
	+20	16.7481	277.357	138.288	16.7676	286.944	142.108

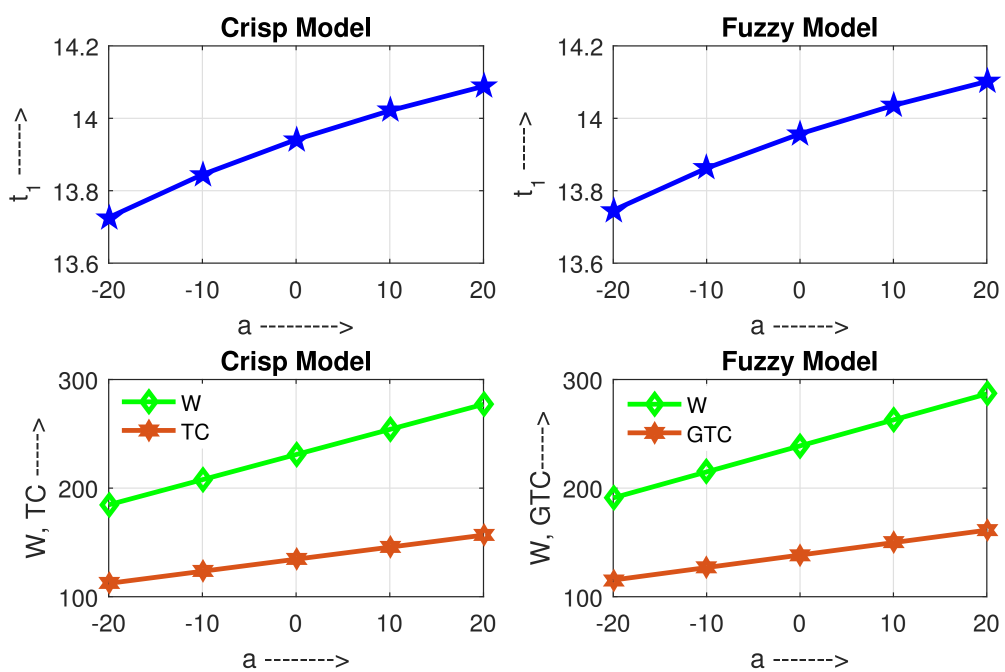

FIGURE 10. Figures of a

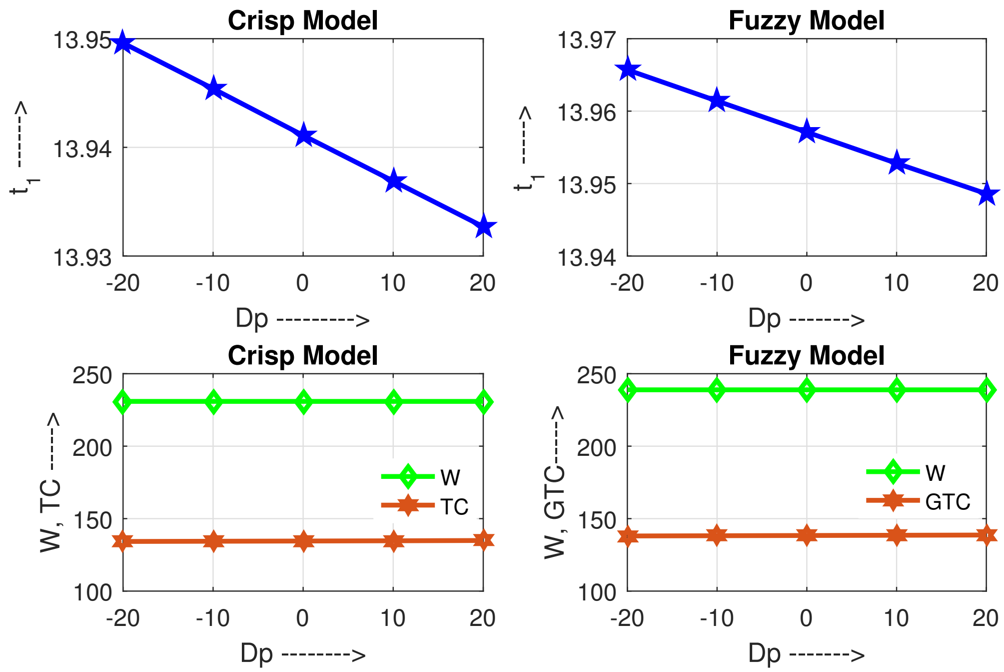

FIGURE 11. Figures of Dp

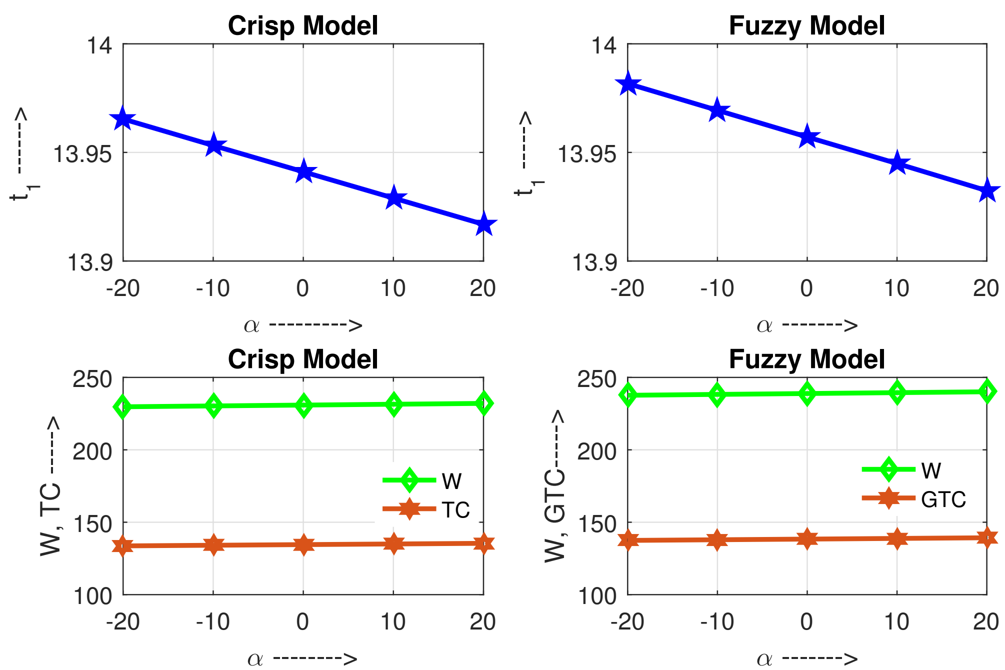

FIGURE 12. Figures of α

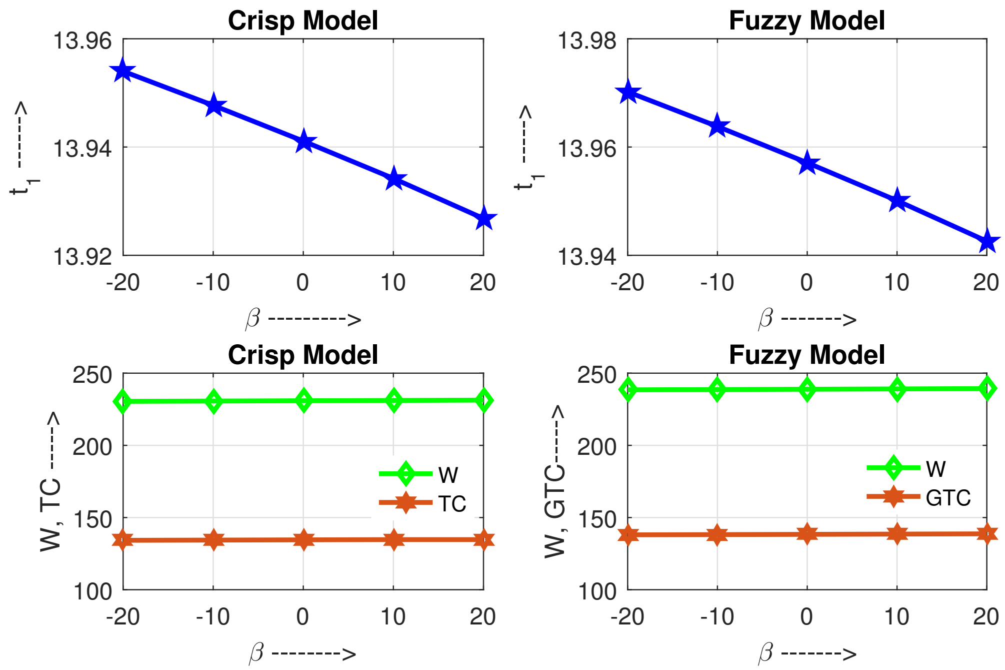

FIGURE 13. Figures of β

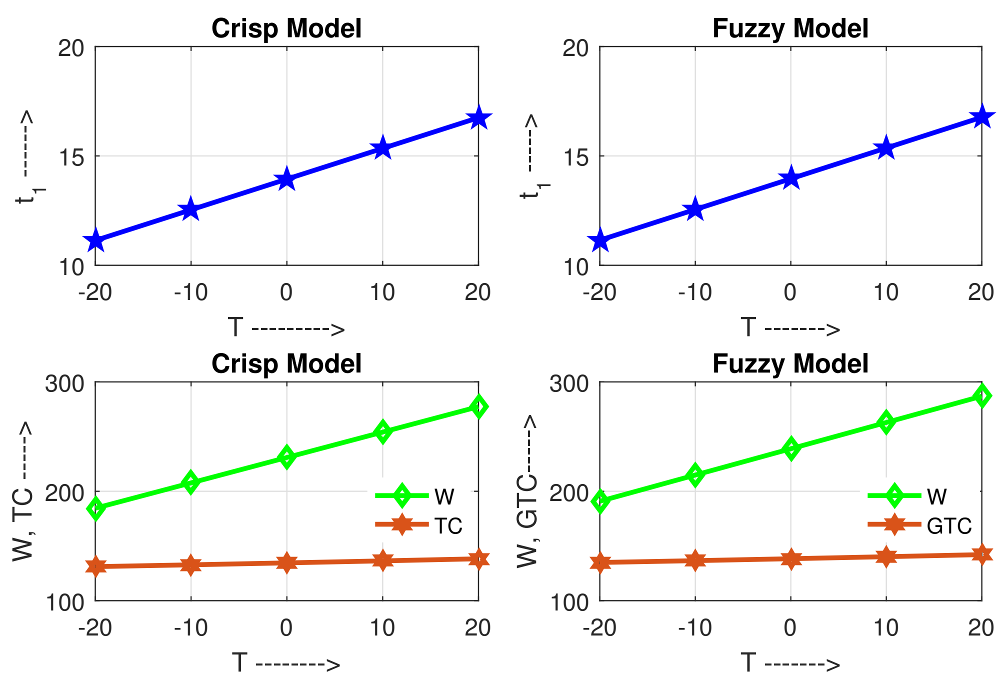

FIGURE 14. Figures of T

Managerial insights.

The sensitivity analysis of some significant key parameters revealed the following managerial insights. From TABLE 1, it is noticed that the change in the value of

(i) demand parameter (a) influences the optimal strategy of inventory significantly (see FIGURE 10). In particular, a decrease in the demand parameter (a) implies the decrease of positive inventory time (t_1), order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC). Moreover, an increase in the demand parameter (a) leads to the increase of positive inventory time (t_1), order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC). Thus, the inventory managers can make a small decrease in the demand parameter in order to minimize the overall cost.

(ii) deterioration cost (D_p) influences the optimal strategy of inventory significantly (see FIGURE 11). In particular, a decrease in the deterioration cost (D_p) implies the increase of the positive inventory time (t_1) and order quantities ($\mathbb{W}$). However, overall cost functions (TC and GTC) decrease. Moreover, an increase in the deterioration cost (D_p) leads to the decrease of the positive inventory time (t_1) and order quantities ($\mathbb{W}$), whereas overall cost functions (TC and GTC) increase. Thus, the inventory managers may try to reduce the value of deterioration cost to minimize the overall cost.

(iii) scale parameter (α) influences the optimal strategy of inventory significantly (see FIGURE 12). In particular, a decrease in the scale parameter (α) implies the increase of the positive inventory time (t_1). However, order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC) decrease. Moreover, an increase in the scale parameter (α) leads to the decrease of the positive inventory time (t_1), whereas order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC) increase. Thus, the inventory managers may try to reduce the value of scale parameter to minimize the overall cost.

(iv) shape parameter (β) influences the optimal strategy of inventory significantly (see FIGURE 13). In particular, a decrease in the shape parameter (β) implies the increase of the positive inventory time (t_1). However, order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC) decrease. Moreover, an increase in the shape parameter (β) leads to the decrease of the positive inventory time (t_1), whereas order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC) increase. Thus, the inventory managers may try to reduce the value of shape parameter to minimize the overall cost.

(v) total cycle length (T) influences the optimal strategy of inventory significantly (see FIGURE 14). In particular, a decrease in the total cycle length (T) implies the decrease of positive inventory time (t_1), order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC). Moreover, an increase in the total cycle length (T) leads to the increase of positive inventory time (t_1), order quantities ($\mathbb{W}$) and overall cost functions (TC and GTC). Thus, the inventory managers can make a small decrease in the total cycle length in order to minimize the overall cost.

9. Conclusion

Through this study, we have developed an optimal EOQ inventory model for deteriorating products with price-sensitive demand and a Weibull deterioration rate, while considering complete backlogged shortages. The model incorporates crucial factors such as demand elasticity, deterioration, and backlogging to better mirror real-world inventory challenges. By examining the model in both crisp and fuzzy environments, the study accounts for the uncertainties in inventory parameters commonly encountered in practice. In the crisp model, where parameters are treated as fixed values, precise optimization of inventory costs is achieved, providing a reliable basis for decision-making in stable conditions. In contrast, the fuzzy model utilizes triangular fuzzy numbers and defuzzification via the graded mean integration representation method, offering a more adaptable and effective approach for handling imprecise parameters, particularly in uncertain situations. The results demonstrate that the fuzzy model delivers greater flexibility and accuracy in environments where parameters are subject to variability. The sensitivity analysis conducted in this study underscores the influence of key parameters on optimal solutions, providing valuable insights for businesses managing deteriorating goods and price-sensitive demand. The proposed model proves especially beneficial for industries dealing with perishable products, where deterioration and fluctuating demand play a significant role in inventory management.

Overall, this study makes a significant contribution to the field of inventory management by offering a more comprehensive framework that addresses the complexities of deterioration, demand elasticity, and backlogging, while also accounting for the uncertainties typically encountered in real-world operations. Future research could expand on this work by exploring additional types of uncertainty, refining defuzzification techniques, and incorporating more sophisticated demand functions, such as pricing, stock levels, displayed stock demand, advertisement-driven demand, ramp-type demand, and others. This would further enhance the model's applicability across various industries. Moving forward, potential areas for future research include dynamic pricing strategies, multi-echelon supply chain structures, investments in appropriate preservation technologies, the use of interval methods or stochastic elements, diverse learning techniques, and the integration of machine learning algorithms.

Statements and Declarations

The authors declare that there are no conflicts of interest associated with this study. Additionally, the research did not receive any funding.

Acknowledgments

This paper is dedicated to Professor Hari Mohan Srivastava on the occasion of his $85th$ birthday.

References

[1] S. Barik, S. Mishra, S. K. Paikray, and U. K. Misra. A deteriorating inventory model with shortages under price dependent demand and inflation. *Asian Journal of Mathematics and Computer Research*, 2016:14–25, 2016.

[2] S. Barik, S. K. Paikray, and U. K. Misra. Inventory model of deteriorating items for nonlinear holding cost with time dependent demand. *Journal of Advances in Mathematics*, 9:2705–2709, 2014.

[3] S. Barik, S. Mishra, S. K. Paikray, and U. K. Misra. An inventory model for deteriorating items under time varying demand condition. *International Journal of Applied Engineering Research*, 10:35770–35773, 2015.

[4] T. H. Burwell, D. S. Dave, K. E. Fitzpatrick, and M. R. Roy. Econimic lot size model for price dependent demand under quality and freight discounts. *International Journal of Production Economics*, 48:141–155, 1997.

[5] S. H. Chen and C. H. Hsieh. Graded mean integration representation of generalized fuzzy number. *Journal of Chinese Fuzzy Systems*, 5:1–7, 1999.

[6] S. K. Indrajitsingha, P. N. Samanta, and U. K. Misra. A fuzzy two-warehouse inventory model for single deteriorating item with selling-price-dependent demand and shortage under partial-backlogged condition. *Applications and Applied Mathematics: An International Journal*, 14:511-536, 2019.

[7] S. K. Indrajitsingha, S. S. Routray, S. K. Paikray, and U. Misra. Fuzzy economic production quantity model with time dependent demand rate. *LogForum*, 12:193–198, 2016.

[8] S. K. Indrajitsingha, A. K.Sahoo, P. N. Samanta, U. K.Misra, and L. K. Raju. Fuzzy EOQ inventory model for price-dependent demand of deteriorating items. *Indian Journal Of Natural Sciences*, 11:28868–28877, 2020.

[9] C. K. Jaggi, P. Gautam, and A. Khanna. Inventory decisions for imperfect quality deteriorating items with exponential declining demand under trade credit and partially backlogged shortages. In P. Kapur, U. Kumar and A. Verma, editors, *Quality, IT and Business Operations Modeling and Optimization. Springer Proceedings in Business and Economics*, pages 213–229, 2018. Springer, Singapore.

[10] A. K. Jalan and K. S. Choudhuri. An EOQ model for deteriorating items in a decling market with SFI policy. *Korean Journal of Computational & Applied Mathematics*, 6:437–449, 1999.

[11] M. Y. Jani, M. R. Betheja, U. Chaudhari, and B. Sarkar. Optimal investment in preservation technology for variable demand under trade-credit and shortages. *Mathematics*, 9:1301, 2021.

[12] B. A. Kumar, S. K. Paikray, S. Mishra, and S. S. Routray. A fuzzy inventory model of defective items under the effect of inflation with trade credit financing. In O. Castillo, D. Jana, D. Giri and A. Ahmed, editors, *Recent Advances in*

Intelligent Information Systems and Applied Mathematics, ICITAM 2019, Studies in Computational Intelligence, vol 863, pages 804–821, 2019. Springer, Switzerland.

[13] B. A. Kumar, S. K. Paikray, and H. Dutta. Cost optimization model for items having fuzzy demand and deterioration with two-warehouse facility under the trade credit financing. *AIMS Mathematics*, 5:1603–1620, 2020.

[14] B. A. Kumar, S. K. Paikray, and U. Misra. Two-storage fuzzy inventory model with time dependent demand and holding cost under acceptable delay in payment. *Mathematical Modelling and Analysis*, 25:441–460, 2020.

[15] B. A. Kumar and S. K. Paikray. Cost optimization inventory model for deteriorating items with trapezoidal demand rate under completely backlogged shortages in crisp and fuzzy environment. *RAIRO-Operations Research*, 56:1969–1994, 2022.

[16] B. A. Kumar, S. K. Paikray, and B. Padhy. Retailer's optimal ordering policy for deteriorating inventory having positive lead time under pre-payment interim and post-payment strategy. *International Journal of Applied and Computational Mathematics*, 8:165, 2022.

[17] S. Kumar and U. S. Rajput. Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. *Applied Mathematics*, 6:496–509, 2015.

[18] U. Mishra, L. E. C. Barrón, S. Tiwari, A. A. Shaikh, and G. T. Garza. An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. *Annals of Operations Research*, 254:165–190, 2017.

[19] U. Mishra, J. T. Aguilera, S. Tiwari and L. E. Cárdenas-Barrón. Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments. *Mathematical Problems in Engineering*, 2018:Article ID 6962417, 2018.

[20] U. Mishra, J. Z. Wu, and M. L. Tseng. Effects of a hybrid-price-stock dependent demand on the optimal solutions of a deteriorating inventory system and trade credit policy on re-manufactured product. *Journal of Cleaner Production*, 241:Article ID 118282, 2019.

[21] B. Mondal, A. K. Bhunia, and M. Maiti. An inventory system of ameliorating items for price dependent demand rate. *Computers & Industrial Engineering*, 45:443–456, 2023.

[22] D. K. Nayak, S. S. Routray, S. K. Paikray and H. Dutta. A fuzzy inventory model for Weibull deteriorating items under completely backlogged shortages. *Discrete and Continuous Dynamical Systems - S*, 14:2435–2453, 2021.

[23] D. K. Nayak, S. K. Paikray, and A. K. Sahoo. A fuzzy inventory model of deteriorating items with time-dependent demand under permissible delay in payment. In H. Dutta, N. Ahmed, and R. P. Agarwal, editors, *Applied Nonlinear Analysis and Soft Computing, ANASC 2020. Advances in Intelligent Systems and Computing*, vol 1437, pages 77–106, 2023. Springer, Singapore.

[24] D. K. Nayak, S. K. Paikray, and A. K. Sahoo. A fuzzy EOQ model for deteriorating items having time dependent demand under partially backlogged shortages. *Proceedings of the Jangjeon Mathematical Society*, 26:269–290, 2023.

[25] D. K. Nayak, S. K. Paikray, and A. K. Sahoo. Profit maximization inventory model for non-instantaneous deteriorating items with imprecise costs. In J. Nayak, S. Varshney, and C. Shekhar, editors, *Modeling and Applications in Operations Research*, pages 123–138, 2024. CRC Press.

[26] D. K. Nayak, A. K. Sahoo, and S. K. Paikray. A deteriorating inventory model with amelioration under power pattern demand in crisp and fuzzy environments. *Mathematical Foundations of Computing*, 8:599–617, 2025.

[27] B. Padhy, P. N. Samanta, S. K. Paikray, and U. K. Misra. An EOQ model for items having fuzzy amelioration and deterioration. *Applied Mathematics & Information Sciences*, 16:353–360, 2022.

[28] S. K. Paikray, S. Misra, M. Mallick, and U. K. Misra. An EOQ Model for both ameliorating and deteriorating items under the influence of inflation and time value of money. *Journal of Computations and Moddelling*, 1:101–113, 2011.

[29] S. K. Paikray, S. Misra, U. K. Misra, and S. Barik. Optimal control of an inventory system for weibull ameliorating, deteriorating items under the influence of inflation. *Bulletin of Pure and Applied Sciences*, 30:85–94, 2011.

[30] S. K. Paikray, S. Misra, U. K. Misra, and S. Barik. An inventory model for inflation indiced demand and Weibull deteriorating items. *International Journal of Advances in Engineering & Technology*, 4:176–182, 2012.

[31] S. K. Patra, S. K. Paikray, and H. Dutta. An inventory model under power pattern demand having trade credit facility and preservation technology investment with completely backlogged shortages. *Journal of Industrial and Management Optimization*, 20:2652–2679, 2024.

[32] S. K. Patra, S. K. Paikray, and B. A. Kumar. A retailer's deteriorating inventory model with amelioration and permissible backlogging under power pattern demand. *Journal of the Indian Society for Probability and Statistics*, 25:597–619, 2024.

[33] S. K. Patra, S. K. Paikray, and R. M. Tripathy. A retailer's inventory model for deteriorating items under power pattern demand with shortages partially backlogged in both crisp and fuzzy environments. *International Journal of Industrial and Systems Engineering*, 49:96–118, 2025.

[34] S. S. Routray, S. K. Paikray, S. Mishra, and U. Misra. A model on deteriorating items with price dependent demand rate. *International Journal of Recent Research and Applied Studies*, 25:44-49, 2015.

[35] S. S. Routray, S. K. Paikray, S. Mishra, and U. K. Misra. Fuzzy inventory model with single item under time dependent demand and holding cost. *International Journal of Advanced Research*, 6:1604–1618, 2017.

[36] S. Saha. Fuzzy inventory model for deteriorating items in a supply chain system with price dependent demand and without backorder. *American Journal of Engineering Research*, 6:183–187, 2017.

[37] I. Sangal, A. Agarwal, and S. Rani. A fuzzy environment inventory model with partial backlogging under learning effect. *International Journal of Computer Applications*, 137:25–32, 2016.

[38] B. Sarkar. An EOQ model with delay in payment and time varying deterioration rate. *Mathematical and Computer Modelling*, 55:367–377, 2012.

[39] B. Sarkar, S. Saren, and H. Wee. An inventory model with variable demand, component cost and selling price for deteriorating items. *Economic Modelling*, 30:306–310, 2013.

[40] N. Sen, B. K. Nath, and S. Saha. A fuzzy inventory model for deteriorating items based on different defuzzification techniques. *American Journal of Applied Mathematics and Statistics*, 6:128–137, 2016.

[41] A. A. Shaikh, L. E. Cárdenas-Barrón, A. K. Bhunia, and S. Tiwari. An inventory model of a three parameter weibull distributed deteriorating item with variable demand dependent on price and frequency of advertisement under trade credit. *RAIRO-Operations Research*, 53:903–916, 2019.

[42] D. Sharmila and R. Uthayakumar. Inventory model for deteriorating items involving fuzzy with shortages and exponential demand. *International Journal of Supply and Operations Management*, 2:888–904, 2015.

[43] P. S. You. Inventory policy for products with price and time-dependent demands. *Journal of the Operational Research Society*, 56:870–873, 2005.

[44] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[45] H. J. Zimmermann. *Fuzzy Set Theory - and Its Applications*. Springer Dordrecht, New York, 2011.