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SMOOTHED FMSG ALGORITHM FOR SOLVING DOCK-DOOR ASSIGNMENT PROBLEM

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Abstract. Cross-docking is a storage process in which products from different companies are combined in a warehouse according to their shipping locations and shipped within a short time. One of the main problems in cross-docking is the assignment of trucks to doors. For this reason, this problem is frequently addressed in the literature. However, in these studies, it is generally assumed that the number of workers assigned to the doors and the service mode of the doors are known. In this study, different from the literature, an integrated problem is addressed in which the trucks to be assigned to the doors, the service modes of the doors and the number of workers to be assigned to the doors are decided simultaneously. A mixed integer nonlinear programming (MINLP) model and a smoothed Feasible Value Based Modified Subgradient (F-MSG) algorithm are developed to solve the integrated problem. F-MSG algorithm solves the sharp augmented Lagrangian dual problems, where zero duality gap property is guaranteed for a wide class of optimization problems without convexity assumption. F-MSG algorithm has no requirements on the type of a norm term used in the sharp augmented Lagrangian. In this paper, to formulate a dual problem, we use the sharp augmented Lagrangian with ℓ_1 norm term. We change the norm term so that the new formulation becomes smoothed and utilize the so-obtained version of the F-MSG algorithm. The performance of the smoothed version of F-MSG algorithm is demonstrated by using test instances taken from the literature. The obtained results demonstrate the strength of the applied modification on the mathematical model.

Keywords. Dock-door assignment problem, Smoothed FMSG algorithm.

1. Introduction

Cross-docking centers are distribution centers where products from suppliers are unloaded, unloaded products are brought together according to the demands of the customers to be sent, and sent to customers without storage or after being stored for a very short time. When a truck sent from supplier companies arrives at the cross-docking center, it is assigned to a door so that it can unload its products. The loads are unloaded at this door and classified according to the customers to whom they will be sent. Each customer's product group is transported to the door to which the truck that will go to that customer is assigned. When all the products to be sent to a customer are completed, the products are loaded onto the truck and the shipment is carried out. Doors in a cross-docking center can operate in different service modes. Some of the most common modes in these facilities are exclusive and mixed. In the exclusive service mode, the doors,which provide a single type of service, are either dedicated only to the unloading operations of trucks coming from suppliers or to the loading operations of trucks going to customers. In the mixed service mode, all doors can serve both incoming and outgoing trucks. In cross-docking centers, the decision on which doors the trucks are assigned to, greatly affects the

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transportation during the transfer of unloaded loads to the shipping doors. Therefore, the problem of door assignment, which determines which doors the incoming and outgoing trucks are assigned to, has been frequently addressed in the literature.

In the cross-docking literature, it is generally assumed that the number of workers assigned to the door is certain and only the focus is made on the assignment of trucks to the doors. However, the number of workers assigned to a door is a factor that greatly affects truck loading and unloading times. Therefore, it is important to consider. Table 1 presents the studies that address the door assignment problem in cross-docking. The second and third columns of the table indicate whether they consider the workforce (W) and the considered service mode (SM). The last two columns provide the objective function and solution method of the relevant studies, respectively.

TABLE 1. Studies on Cross-Docking Door Assignment Problem

Study	W	SM	Objective	Solution Method
Tarhini et al. [33]		Excl.	total material handling distance	SS
Oh et al. [25]		Excl.	total material handling distance	MINLP,H,GA
Gelareh et al. [12]		Excl.	total transportation cost	MIP
Li et al. [19]		Excl.	total transportation cost	H
Enderer et al. [5]		Excl.	total material handling and transportation cost	MIP,CG
Escudero et al. [6]		Excl.	total transportation cost	MIP,H
Nassief et al. [22]		Excl.	total material handling cost	MIP,LR
Nassief et al. [23]		Excl.	total material handling cost	MIP,CG
Wang and Alidaee [37]		Excl.	total material handling cost	MIP,TS
Zhang et al. [40]		Excl.	total material handling cost	ACO
Essghaier et al. [7]		Excl.	total cost	FCP
Ghomi et al. [13]		Excl.	total cost	MINLP,TS,BLPF
Gallo et al. [8]		Excl.	total cost	GA
Miao et al. [21]		Excl.	total cost	MIP,GA
Wisittipanich and Hengmeechai [38]		Excl.	total earliness and tardiness	MIP,MODE
Ozden and Saricicek [27]		Excl.	total earliness and tardiness	SA,TS
Acar et al. [1]		Excl.	total square of the slack times	MINLP,H
Sayed et al. [29]		Excl.	makespan	MIP,HA
Dondo and Cerdá [4]		Excl.	makespan,total cost	MIP
Van Belle et al. [36]		Excl.	total travel time and total tardiness	MIP,TS
Konur and Golias [17]	✓	Excl.	the total labor cost	GA
Tadumadze et al. [32]	✓	Excl.	total flow time or punctuality	MIP,H
Xi et al. [39]		Mixed	total cost and number of conflicts	CCG
Li et al. [20]		Mixed	total process time	GA,HA
Shakeri et al. [30]		Mixed	makespan	H
Hermel et al. [15]		Mixed	makespan	FSSA
Rijal et al. [28]		Mixed	total cost	ALNS
Neamatian Monemi et al. [24]		Mixed	total cost	B&C
This study	✓	Mixed	total time,total number of workers	LF-MSG

W: Worker, TS: Truck Scheduling, MP: Multi-Period, SM: Service Mode, SS: Scatter Search, MIP: Mixed Integer Programming Model, MINLP: Mixed-Integer Nonlinear Programming, HA: Hybrid Algorithm, CG: Column Generation, CCG: A column and constraint generation algorithm, GA: Genetic Algorithm, H: Heuristic Algorithm, FCP: Fuzzy Chance Programming, FSSA: Four Stage Solution Approach. LR: Lagrangian Relaxation, ALNS: adaptive large neighborhood search, MODE: Multi-Objective Differential Evolution, SA: Simulated Annealing, TS: Tabu Search, BLPF: Best-Local-Point-Finder Algorithm, ACO: Ant Colony Optimization, B&C: Branch and Cut, LF-MSG: Linearized Feasible value based Modified Subgradient

Considering the related literature, mixed mode is discussed in a small number of studies. Similarly, the number of workers that will be assigned to each door has been taken into consideration in very few of the studies. The study dealing with these two important concepts together could not be accessed. Considering the reached literature, this is the first study that considers the door assignment problem in cross-docking with both the mixed mode and the number of workers.

Lagrangian relaxation and subgradient algorithms have been widely applied for solving integer or mixed integer programming problems. However, classic Lagrangian techniques often result in a duality gap and generally cannot determine the optimum value of the primal integer optimization problems such as the quadratic 0-1 problems that are non-convex [18].

Recently, a considerable amount of works have been published on different augmented Lagrangian duality methods that can eliminate the duality gap in most non-convex problems and obtain good solutions. Probably the first efficient method called the Modified Subgradient (MSG) Method, for solving general nonconvex constrained optimization problems, was developed by Gasimov [9]. Gasimov proved that this method generates strongly onotonically increasing sequence of dual values which converges to the common primal-dual optimal value. The version of MSG method was later developed by Kasimbeyli et.al. Kasimbeyli et al. [16], where no exact global minimum of the augmented Lagrangian function was required to update dual variables at each iteration. The authors applied this algorithm for solving different kind of optimization problems , see e.g. Gasimov and Rubinov [10] . Gasimov and Ustun [11] demonstrated the performance of the MSG for non-convex 0-1 quadratic assignment problems.

In another study, Sipahioglu and Saraç [31] examined the performance of the algorithm for QKP with an inequality constraint. To solve a general portfolio optimization problem, Ustun and Kasimbeyli [35] applied the feasible value-based modified subgradient (F-MSG) algorithm, which is a generalized version of the MSGA. Ulutas and Saraç [34], handled a multi-period facility layout problem where the sum of material handling and re-layout costs are minimized. They proposed the MSGA for this problem and determined its parameters of MSGA by using the design of the experiment. In another study, Ozcelik and Saraç [26] addressed the cell formation problem with alternative part routes to minimize the weighted sum of the voids and the exceptional elements. They proposed a hybrid genetic algorithm based on MSGA. Alpaslan Takan and Kasimbeyli [2] developed a new hybrid subgradient algorithm for solving the capacitated vehicle routing problem. In another recent study conducted by Bulbul and Kasimbeyli [3], a new version of the aircraft maintenance routing problem is addressed. The authors proposed a hybrid solution approach for this problem, which hybridized the F-MSGA and the ant colony optimization metaheuristic. As can be seen from these studies, MSGA is a successful solution method that is widely used in solving discrete problems with linear or quadratic objective functions. However, two difficulties can be encountered when using this algorithm. The first is that solving the dual problem can be very difficult. So, the studies on the MSGA in the literature in which the hybrid solution approach is suggested have generally focused on the solution of the dual problem and used metaheuristic algorithms to solve the dual problem. However, another important issue affecting the performance of the MSGA is the determination of appropriate parameter values. In the literature, only one study [34] has been accessed to determine the parameters of the MSGA by using the experimental design method.

The rest of the paper is organized as follows. In Section 2, we explain the problem definition and the mathematical model. Section 3 presents the proposed solution method. In this section we give definition of the sharp augmented Lagrangian formulation of the associated dual problem and the solution algorithm. Then the smoothed version of the problem is presented. Computational results are given in Section 4. Finally, Section 5 draws some conclusiins and outlines future works.

2. Problem Definition and Proposed Mathematical Model

In the considered problem, there are two types of trucks in a cross-docking system, incoming and outgoing. Incoming trucks carry products to be transferred to outgoing trucks. An outgoing truck can take loads from more than one incoming truck, and an incoming truck can carry loads for more than one outgoing truck. It is predetermined how much load will be transferred from each incoming truck to each outgoing truck. Each truck must be assigned to a door. The total number of workers that can be assigned to the doors for loading and unloading is limited. More than one worker can be assigned to a door. The durations of loading and unloading operations depend on the number of workers assigned to the relevant door. The doors are not dedicated to loading and unloading operations. All doors can provide mixed service. The objective of the problem is to minimize the sum of unloading, transportation, and loading times.

The indices, parameters, decision variables, objective and constraint functions of the proposed mathematical model are given below.

$Indexes$:

$I = \{1, 2, \ldots, \alpha\}$ $\quad i, j \in I$ door
$K = \{1, 2, \ldots, \beta\}$ $\quad k, l \in K$ truck
$W = \{1, 2, \ldots, \delta\}$ $\quad w \in W$ worker number

$Parameters$:

α : number of doors
β : number of trucks
δ : maximum number of workers that can be assigned to a door
g : total number of workers
q_k : type of truck k (0 for incoming truck, 1 for outgoing truck)
t_{ij} : transportation time between doors i and j
u_w : unit unloading time with w workers
a_w : unit loading time with w workers
b_{kl} : amount of load to be transported from truck k to truck l
s_k : total amount of load to be unloaded from truck k ($s_k = \sum_l b_{kl}$)
r_l : total amount of load to be loaded from truck l ($r_l = \sum_k b_{kl}$)
C_i : capacity of door i

$Decision\ Variables$:

x_{ik} : 1, if truck k is assigned to door i; 0, otherwise.
y_{iw} : 1, if w workers are assigned to door i; 0, otherwise.
v_{ijk} : amount of load to be transported from truck k assigned to door i to door j

$Objective\ Function$:

$$min\ z = \sum_i \sum_{k|q_k=0} \sum_w s_k u_w x_{ik} y_{iw} + \sum_i \sum_j \sum_{k|q_k=0} t_{ij} v_{ijk} + \sum i \sum_{k|q_k=1} \sum_w r_k a_w x_{ik} y_{iw} \tag{2.1}$$

$Constraints:$

$$\sum_i x_{ik} = 1 \qquad \forall k \tag{2.2}$$

$$\sum_w y_{iw} \leq 1 \qquad \forall i \tag{2.3}$$

$$\sum_k x_{ik} \leq \beta \sum_w y_{iw} \qquad \forall i \tag{2.4}$$

$$\sum_k q_k r_k x_{ik} + \sum_k (1-q_k) s_k x_{ik} \leq C_i \qquad \forall i \tag{2.5}$$

$$\sum_i \sum_w w y_{iw} \leq g \tag{2.6}$$

$$\sum_i v_{ijk} = s_k x_{ik} \qquad \forall k \,|\, q_k = 0, \forall i \tag{2.7}$$

$$\sum_i v_{ijk} = \sum_l b_{kl} x_{jl} \qquad \forall k \,|\, q_k = 0, \forall j \tag{2.8}$$

$$x_{ik} \in \{0,1\} \qquad \forall i, \forall k \tag{2.9}$$

$$y_{iw} \in \{0,1\} \qquad \forall i, \forall w \tag{2.10}$$

$$w_{ijk} \geq 0 \qquad \forall i, \forall j, \forall k \tag{2.11}$$

$$\tag{2.12}$$

The objective (2.1) is to minimize the sum of unloading, transportation, and loading times. Equation (2.2) guarantees that each truck is assigned to a door. Equation (2.3) determines the number of workers to be assigned to each door. Equation (2.4) prevents the assignment of any truck to a door if no workers are assigned to it. Equation (2.5) is the capacity constraints of the doors. Equation (2.6) limits the total number of assigned workers. Equations (2.7) and (2.8) are flow conservation constraints for doors. Equations (2.9)-(2.11) are relationship constraints among decision variables. Equations (2.9)-(2.11) are sign constraints.

3. Proposed Solution Approach

3.1. Smoothed version of MSG algorithm.

Let the primal problem P be given as follows,

$$\min P = \min_{x \in S} f(x)$$
$$subject\ to\ g_i(x) = 0, i = 1, .., m$$

where,S is a compact subset of a metric space X, and $f : X \to R$ and $g : X \to R^n$ are given functions. The sharp augmented Lagrangean function $L : S \times R^n \times R_+$ associated with P:

$$L(x, u, c) = f(x) + c\|g_i(x)\| - \langle g_i(x), u\rangle$$

where, c and u are the dual variables, $\|.\|$ is the norm and $\langle ., . \rangle$ is the inner product on R^n.

The dual function $H : R^n \times R_+ \to R$ associated with the problem P is defined as

$$H(u, c) = \min_{x \in S} L(x, u, c), for\ u \in R^n, and\ c \in R_+$$

Then the dual problem P^* is given by:

$$\max_{(u,c)\in R^n\times R_+} H(u,c)$$

The F-MSG algoirthm developed by Kasimbeyli et al. [16] is given below:

$Initialization\ Step$: Choose a vector $(u_1, c_1) \in R^n \times R_+$. Let $k = 1$.

$Step\ 1$: Given Lagrange multipliers (u_k, c_k), solve the following sub problem:

$$\begin{aligned} &\underset{x\in S}{minimize} \quad f(x) + c_k\|g(x)\| - \langle u_k, g(x)\rangle \\ &subject\ to \quad f(x) + c_k\|g(x)\| - \langle u_k, g(x)\rangle \le \bar{H} \end{aligned} \tag{3.1}$$

Let x_k be the global solution of this problem. If $\|g(x)\| = 0$ then $stop$. (u_k, c_k) is a solution to the dual problem (P^*), x_k is a solution to (P). Otherwise, go to $Step\ 2$.

$Step\ 2$: Let $u_{k+1} = u_k - s_k g(x_k), c_{k+1} = c_k + (s_k + \varepsilon_k)\|g(x_k)\|$, where s_k and ε_k are positive scalar step sizes, replace k by $k+1$ and repeat $Step\ 1$.

The step size parameters s_k and ε_k are defined as follows:

$$s_k = \frac{\delta\alpha\left(\bar{H} - L(x_k, u_k, c_k)\right)}{(\alpha^2 + (1+\alpha)^2)\|g(x_k)\|^2},$$

$0 < \varepsilon_k < s_k$, where $\bar{H}$ is an approximate optimal value or an upper bound for the dual problem, $\alpha > 0$ and $0 < \delta < 2$.

In these algorithms the following sub-problem (3.1) is solved.Instead of the sub-problem (3.1), we used the following sub-problem (3.2) by using the smoothed form of the l_1-norm.

$$\begin{aligned} &\underset{x\in S}{Minimize} \quad f(x) - \sum_{i=1}^{m} u_{ki} g_i(x) + c_k \sum_{i=1}^{m} (z_i^+ + z_i^-) \\ &subject\ to \quad f(x) - \sum_{i=1}^{m} u_{ki} g_i(x) + c_k \sum_{i=1}^{m} (z_i^+ + z_i^-) \le \bar{H} \\ &\qquad g_i(x) - z_i^+ + z_i^- = 0 \quad i = \overline{1,m} \\ &\qquad z_i^+ z_i^- = 0 \\ &\qquad z_i^+, z_i^- \ge 0 \end{aligned} \tag{3.2}$$

where $z_i^+ = \begin{cases} g_i(x) & g_i(x) \ge 0 \\ 0 & g_i(x) < 0 \end{cases}, \quad z_i^- = \begin{cases} -g_i(x) & g_i(x) < 0 \\ 0 & g_i(x) \ge 0 \end{cases},$
$z_i^+ + z_i^- = |g_i(x)|$ and $z_i^+ - z_i^- = g_i(x)$.

4. Computational Results

To test the proposed solution approachs, 5 sample problems taken from the study of Guignard et al. [14] were used.

In all problems, maximum 5 workers $(0 \le \delta \le 5)$ can be assigned to a door. In the case when $\delta = 0$, the door is considered out of service. The total number of workers (g) is calculated by using the following formula:

$$g = \alpha \left\lfloor \frac{\delta}{2} \right\rfloor \tag{4.1}$$

The unit processing times depending on the number of workers are given in equations (4.2) and (4.3) for unloading and loading, respectively.

$$u_w = \begin{cases} u_1 & w = 1 \\ 0.7u_{w-1} & w > 1 \end{cases} \tag{4.2}$$

$$a_w = \begin{cases} a_1 & w = 1 \\ 0.7a_{w-1} & w > 1 \end{cases} \tag{4.3}$$

The total amount of doors that can be used for both loading and unloading equal to 8. The transportation times between doors are given in Table 2.

TABLE 2. Values of t_{ij} parameters

i/j	1	2	3	4	5	6	7	8
1	0	1	2	3	8	9	10	11
2	1	0	1	2	9	8	9	10
3	2	1	0	1	10	9	8	9
4	3	2	1	0	10	10	9	8
5	8	9	10	10	0	1	2	3
6	9	8	9	10	1	0	1	2
7	10	9	8	9	2	1	0	1
8	11	10	9	8	3	2	1	0

Sample problems are solved with both GAMS/Dicopt and FMSG algorithms. Below, all parameter values and obtained solutions for each sample are presented in detail.

4.1. Sample 1.

In the Sample 1, there are 8 incoming trucks, 8 outgoing trucks. The unloading (u_1) and loading (a_1) times with one worker are 9 and 11, respectively. The capacities of all doors are 159. The amount of load to be transported between incoming and outgoing trucks is given in Table 3.

TABLE 3. Values of b_{kl} parameters

k/l	9	10	11	12	13	14	15	16
1	0	0	26	0	0	0	0	0
2	22	0	0	0	0	0	0	0
3	41	32	0	50	30	0	0	0
4	0	0	0	0	10	31	0	0
5	40	0	0	44	0	0	50	0
6	0	47	0	31	0	0	0	0
7	0	0	0	0	43	0	0	0
8	0	44	31	0	0	0	0	31

The best value for the objective function objective by using GAMS/Dicopt is 7585. The objective value obtained with the F-MSG algorithm is 7549, which is better than the one obtained with GAMS. The solution results obtained with FMSG algorithm for Sample 1, are given in Table 4.

TABLE 4. Results obtained with FMSG for the sample problem

doors	trucks	service mode	number of workers	loads
1	6, 11	mixed	2	135
2	10, 16	unloading	4	154
3	4, 8	loading	2	147
4	7, 13, 14	mixed	3	157
5	1, 12	mixed	3	151
6	3	loading	3	153
7	2, 5	loading	3	156
8	9, 15	unloading	4	153

4.2. **Sample 2.**

Sample 2 is created by increasing the capacity value of Sample 1 to 174. The objective function value is obtained as 7316 with GAMS/Dicopt and 7299 with FMSG algorithm. The results of FMSG algorithm for Sample 2 are given in Table 5.

TABLE 5. Results obtained with FMSG for the sample problem

doors	trucks	service mode	number of workers	loads
1	12	unloading	3	125
2	3	loading	3	153
3	2, 5	loading	3	156
4	9, 15	unloading	4	153
5	6, 13	mixed	3	161
6	7, 10	mixed	3	166
7	8, 16	mixed	2	137
8	1, 4, 11, 14	mixed	3	155

4.3. **Sample 3.**

Sample 3 is created by increasing the capacity value of Sample 1 to 196. No feasible solution is found with GAMS/Dicopt. The objective function value is obtained as 6179 with FMSG algorithm. The results of FMSG algorithm for Sample 3 are given in Tables 6.

TABLE 6. Results obtained with FMSG for the sample problem

doors	trucks	service mode	number of workers	loads
1	1, 6	loading	1	104
2	4, 10, 14	mixed	4	195
3	8, 11, 16	mixed	4	194
4	-	-	-	-
5	3	loading	3	153
6	9, 13	unloading	4	186
7	2, 7, 12	mixed	4	190
8	5, 15	mixed	4	184

4.4. **Sample 4.**

In the Sample 4, there are 9 incoming trucks, 9 outgoing trucks. The unloading (u_1) and loading (a_1) times with one worker are 10 and 12, respectively. The capacities of all doors are 201. The amount of load to be transported between incoming and outgoing trucks is given in Table 7.

TABLE 7. Values of b_{kl} parameters

k/l	10	11	12	13	14	15	16	17	18
1	0	46	39	0	0	0	0	0	0
2	0	0	49	0	42	0	24	29	0
3	0	0	34	0	0	0	31	0	46
4	0	19	0	0	0	24	0	0	0
5	47	0	14	0	0	0	0	0	0
6	19	48	0	0	0	0	21	0	47
7	0	32	0	0	0	0	0	0	40
8	0	0	0	0	0	0	0	0	26
9	0	0	0	21	0	0	0	0	0

No feasible solution is found with GAMS/Dicopt. The objective function value is obtained as 7002 with FMSG algorithm. The results of FMSG algorithm for Sample 4 are given in Table 8.

TABLE 8. Results obtained with FMSG for the sample problem

doors	trucks	service mode	number of workers	loads
1	-	-	-	-
2	9,10	mixed	1	66
3	5, 12	mixed	5	197
4	2, 14, 17	mixed	4	186
5	11, 13, 15	unloading	4	190
6	1, 4, 7	loading	4	200
7	6, 8, 18	mixed	2	161
8	3, 16	mixed	4	187

4.5. **Sample 5.**

Sample 5 is created by increasing the capacity value of Sample 4 to 210. The objective function values obtained with GAMS/DICOPT and FMSG algorithm, are 5711 and 5629, respectively. The results obtained by FMSG algorithm for Sample 5 are given in Table 9.

TABLE 9. Results obtained with FMSG for the sample problem

doors	trucks	service mode	number of workers	loads
1	-	-	-	-
2	4, 12, 15	mixed	5	203
3	2, 14, 17	mixed	3	186
4	-	-	-	-
5	5, 10, 16	mixed	4	203
6	1, 3	loading	3	196
7	6, 7, 9, 18	mixed	5	207
8	8, 11, 13	mixed	4	192

The objective function values (z) and solution times (t) in seconds obtained with both GAMS/Dicopt and FMSG algorithms for all samples are summarized in Table 10.

Table 10. Summary of Test Results

Sample Problem	*GAMS/Dicopt* z	t	*FMSG* z	t
Sample 1	7585	21	7549	56
Sample 2	7316	224	7299	4400
Sample 3	-	-	6179	873
Sample 4	-	-	7002	3393
Sample 5	5711	67	5629	2361

As can be seen from Table 10, a solution was found with the FMSG algorithm for Samples 3 and 4, for which no feasible solution could be found with GAMS/Dicopt. For Samples 1, 2 and 5 better objective function values were obtained with the FMSG algorithm than GAMS/Dicopt.

5. Conclusion

In this study, we addressed an integrated dock-door assignment problem by simultaneously determining the truck-to-door assignments, the service modes of the doors, and the number of workers allocated to each door. Unlike existing studies that assume predefined service modes and workforce distributions, our approach provides a more realistic and flexible framework for cross-docking operations.

To effectively solve this complex problem, we developed a mixed integer nonlinear programming model and introduced a Smoothed Feasible Value Based Modified Subgradient algorithm. Our approach leverages the sharp augmented Lagrangian dual method with an ℓ_1 norm term, ensuring optimality without requiring convexity assumptions.

The experimental results, obtained from benchmark test instances, demonstrate the effectiveness and superiority of the proposed smoothed F-MSG algorithm. The findings highlight the importance of integrating multiple decisions in dock-door assignment and offer valuable insights for logistics practitioners and researchers aiming to enhance operational efficiency in supply chain management. Future research may explore the extension of this approach to dynamic environments or incorporate uncertainty factors such as fluctuating arrival times and demand variations.

Statements and Declarations

The authors declare that they have no conflict of interest, and the manuscript has no associated data.

References

[1] K. Acar, A. Yalcin, and D. Yankov. Robust door assignment in less-than-truckload terminals. *Computers & Industrial Engineering*, 63(4):729–738, 2012.

[2] M. Alpaslan Takan and R. Kasimbeyli. A hybrid subgradient method for solving the capacitated vehicle routing problem. *Journal of Nonlinear and Convex Analysis*, 21:413–423, 2020.

[3] K. G. Bulbul and R. Kasimbeyli. Augmented lagrangian based hybrid subgradient method for solving aircraft maintenance routing problem. *Computers & Operations Research*, 132:Article ID 105294, 2021.

[4] R. Dondo and J. Cerdá. The heterogeneous vehicle routing and truck scheduling problem in a multi-door cross-dock system. *Computers & Chemical Engineering*, 76:42–62, 2015.

[5] F. Enderer, C. Contardo, and I. Contreras. Integrating dock-door assignment and vehicle routing with cross-docking. *Computers & Operations Research*, 88:30–43, 2017.

[6] L. F. Escudero, M. A. Garín, and A. Unzueta. On solving the cross-dock door assignment problem. *International Journal of Production Research*, 62(4):1262–1276, 2023.

[7] F. Essghaier, H. Allaoui, and G. Goncalves. Truck to door assignment in a shared cross-dock under uncertainty. *Expert Systems with Applications*, 182:Article ID 114889, 2021.

[8] A. Gallo, R. Accorsi, R. Akkerman, and R. Manzini. Scheduling cross-docking operations under uncertainty: A stochastic genetic algorithm based on scenarios tree. *EURO Journal on Transportation and Logistics*, 11:Article ID 100095, 2022.

[9] R. N. Gasimov. Augmented lagrangian duality and nondifferentiable optimization methods in nonconvex programming. *Journal of Global Optimization*, 24(2):187–203, 2002.

[10] R. N. Gasimov and A. M. Rubinov. On augmented lagrangians for optimization problems with a single constraint. *Journal of Global Optimization*, 28(2):153–173, 2004.

[11] R. N. Gasimov and O. Ustun. Solving the quadratic assignment problem using f-msg algorithm. *Journal of Industrial & Management Optimization*, 3(2):173–191, 2007.

[12] S. Gelareh, F. Glover, O. Guemri, S. Hanafi, P. Nduwayo, and R. Todosijević. A comparative study of formulations for a cross-dock door assignment problem. *Omega*, 91:Article ID 102015, 2020.

[13] V. Ghomi, F. Ghazi Nezami, S. Shokoohyar, and M. Ghofrani Esfahani. An optimization model for forklift utilisation and congestion control in cross-docking terminals. *International Journal of Systems Science: Operations & Logistics*, 10(1):Article ID 2142463, 2022.

[14] M. Guignard, P. M. Hahn, A. A. Pessoa, and D. C. da Silva. Algorithms for the cross-dock door assignment problem. In *Proceedings of the Fourth International Workshop on Model-Based Metaheuristics*, pages 145–162, 2012.

[15] D. Hermel, H. Hasheminia, N. Adler, and M. J. Fry. A solution framework for the multi-mode resource-constrained cross-dock scheduling problem. *Omega*, 59:157–170, 2016.

[16] R. Kasimbeyli, O. Ustun, and A. Rubinov. The modified subgradient algorithm based on feasible values. *Optimization*, 58(5):535–560, 2009.

[17] D. Konur and M. M. Golias. Loading time flexibility in cross-docking systems. *Procedia Computer Science*, 114:491–498, 2017.

[18] D. Li. Zero duality gap in integer programming: P-norm surrogate constraint method. *Operations Research Letters*, 25(2):89–96, 1999.

[19] M. Li, J.-K. Hao, and Q. Wu. A flow based formulation and a reinforcement learning based strategic oscillation for cross-dock door assignment. *European Journal of Operational Research*, 312(2): 473–492, 2024.

[20] Y. Li, R.-Y. Tang, L.-W. MuRong, and Q. Sun. Collaborative optimization of dock door assignment and vehicle scheduling in cross-docking. *Journal of the Operations Research Society of China*, 8(3): 493–514, 2019.

[21] Z. Miao, J. Zhang, Y. Lan, and R. Su. A two-stage genetic algorithm for the truck-door assignment problem with limited capacity vehicles and storage area. *Journal of Systems Science and Systems Engineering*, 28(3):285–298, 2019.

[22] W. Nassief, I. Contreras, and R. As'ad. A mixed-integer programming formulation and lagrangean relaxation for the cross-dock door assignment problem. *International Journal of Production Research*, 54(2):494–508, 2015.

[23] W. Nassief, I. Contreras, and B. Jaumard. A comparison of formulations and relaxations for cross-dock door assignment problems. *Computers & Operations Research*, 94:76–88, 2018.

[24] R. Neamatian Monemi, S. Gelareh, and N. Maculan. A machine learning based branch-cut-and-benders for dock assignment and truck scheduling problem in cross-docks. *Transportation Research Part E: Logistics and Transportation Review*, 178:Article ID 103263, 2023.

[25] Y. Oh, H. Hwang, C. N. Cha, and S. Lee. A dock-door assignment problem for the korean mail distribution center. *Computers & Industrial Engineering*, 51(2):288–296, 2006.

[26] F. Ozcelik and T. Saraç. A genetic algorithm extended modified sub-gradient algorithm for cell formation problem with alternative routings. *International Journal of Production Research*, 50(15): 4025–4037, 2012.

[27] G. Ozden and I. Saricicek. Scheduling trucks in a multi-door cross-docking system with time windows. *Bulletin of the Polish Academy of Sciences Technical Sciences*, 67:349–362, 2019.

[28] A. Rijal, M. Bijvank, and R. de Koster. Integrated scheduling and assignment of trucks at unit-load cross-dock terminals with mixed service mode dock doors. *European Journal of Operational Research*, 278(3):752–771, 2019.

[29] S. I. Sayed, I. Contreras, J. A. Diaz, and D. E. Luna. Integrated cross-dock door assignment and truck scheduling with handling times. *TOP*, 28(3):705–727, 2020.

[30] M. Shakeri, M. Y. H. Low, S. J. Turner, and E. W. Lee. A robust two-phase heuristic algorithm for the truck scheduling problem in a resource-constrained crossdock. *Computers & Operations Research*, 39(11):2564–2577, 2012.

[31] A. Sipahioglu and T. Saraç. The performance of the modified subgradient algorithm on solving the 0–1 quadratic knapsack problem. *Informatica*, 20(2):293–304, 2009.

[32] G. Tadumadze, N. Boysen, S. Emde, and F. Weidinger. Integrated truck and workforce scheduling to accelerate the unloading of trucks. *European Journal of Operational Research*, 278(1):343–362, 2019.

[33] A. A. Tarhini, M. M. Yunis, and M. Chamseddine. Natural optimization algorithms for the cross-dock door assignment problem. *IEEE Transactions on Intelligent Transportation Systems*, 17(8): 2324–2333, 2016.

[34] B. Ulutas and T. Saraç. Determining the parameters of msg algorithm for multi period layout problem. *Journal of Manufacturing Technology Management*, 23(7):922–936, 2012.

[35] O. Ustun and R. Kasimbeyli. Combined forecasts in portfolio optimization: A generalized approach. *Computers & Operations Research*, 39(4):805–819, 2012.

[36] J. Van Belle, P. Valckenaers, G. Vanden Berghe, and D. Cattrysse. A tabu search approach to the truck scheduling problem with multiple docks and time windows. *Computers & Industrial Engineering*, 66(4):818–826, 2013.

[37] H. Wang and B. Alidaee. The multi-floor cross-dock door assignment problem: Rising challenges for the new trend in logistics industry. *Transportation Research Part E: Logistics and Transportation Review*, 132:30–47, 2019.

[38] W. Wisittipanich and P. Hengmeechai. A multi-objective differential evolution for just-in-time door assignment and truck scheduling in multi-door cross docking problems. *Industrial Engineering and Management Systems*, 14(3):299–311, 2015.

[39] X. Xi, L. Changchun, W. Yuan, and L. Loo Hay. Two-stage conflict robust optimization models for cross-dock truck scheduling problem under uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, 144:Article ID 102123, 2020.

[40] Y.-H. Zhang, Y.-J. Gong, W.-N. Chen, T.-L. Gu, H.-Q. Yuan, and J. Zhang. A dual-colony ant algorithm for the receiving and shipping door assignments in cross-docks. *IEEE Transactions on Intelligent Transportation Systems*, 20(7):2523–2539, 2019.