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WASTE COLLECTION PROBLEM: MATHEMATICAL MODEL AND SOLUTION METHOD

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Dedicated to Professor Hari Mohan Srivastava on the Occasion of His 85th Birthday

Abstract. This study aims to optimize waste collection processes in the urban district of Eskisehir, Turkey. Increasing urbanization and population density necessitate more efficient and sustainable management of municipal waste collection activities. The study seeks to develop a mathematical model for this problem in the form of a vehicle routing problem (VRP) and a region-specific solution, by utilizing data such as neighborhood-level waste generation volumes, container fill rates, and vehicle capacities. The objective of this research is to develop an optimized routing framework focused on minimizing total operational costs, fuel usage, and the number of deployed vehicles. In alignment with principles of environmental sustainability, the paper aims to provide a practical and adaptable model for local governments. The research incorporates a hybrid framework that combines Genetic Algorithm and Tabu Search—two metaheuristic techniques widely used in the literature. The effectiveness of the proposed hybrid approach is evaluated through computational experiments on the case study, with a comparative analysis of results obtained using OR-Tools. The outcomes of this study aim to offer an innovative perspective on urban waste management and contribute meaningfully to the academic literature.

Keywords. Waste collection, Vehicle routing problem, Mathematical programming, Genetic algorithm, Tabu search.

1. Introduction

The Vehicle Routing Problem (VRP) is a prominent combinatorial optimization problem that focuses on determining the most cost-effective routes for a fleet of vehicles [9]. In typical VRP scenarios, vehicles start from a central depot and are assigned to serve a group of customers while considering vehicle capacity constraints and customer demand. Since the foundational study by Dantzig and Ramser [9], VRP has attracted significant attention in the research community. Various solution approaches have been developed over time, generally categorized into heuristics, metaheuristics, and exact methods, each suited to different problem variants such as open VRP [25], clustered VRP, VRP with time windows, green VRP, and more. Hybrid solution methods have also been widely applied, yielding important results [2].

The Waste Collection Vehicle Routing Problem (WCVRP) has gained growing attention in recent years due to the increasing need for efficient and sustainable waste management solutions. Researchers have proposed diverse methodologies to optimize waste collection routes, considering factors such as environmental impact, operational costs, and service quality. Efficient waste collection and transportation play a crucial role in enhancing overall waste management performance and promoting sustainable development [32].

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The efficiency of urban waste collection has become a critical concern for local governments, particularly with the challenges posed by rapid urbanization and population growth. Today, municipalities are under increasing pressure to optimize their waste collection operations in line with efficient resource utilization and environmental sustainability principles. In densely populated areas, poorly designed routes often result in increased operational costs and elevated carbon emissions, posing both economic and environmental challenges. To address this, effective optimization techniques such as genetic algorithms and tabu search, have been adapted in this study with sensitivity to environmental criteria.

The waste collection process inherently reflects the structure of the VRP. In municipal applications, vehicles depart from a central depot, visit geographically distributed locations (e.g., neighborhoods or bins), and return to the depot, while respecting constraints such as vehicle capacity, service time windows, and route length. This setup closely mirrors the VRP framework, where the objective is to minimize total travel cost or distance while effectively utilizing limited vehicle resources. As a result, VRP-based models have become essential tools in optimizing waste collection strategies to reduce costs, fuel consumption, and environmental impacts. Minimizing cost, distance, or travel time—common objectives in routing problems—also dominate in WCVRP studies. Conversely, efforts to maximize profit, revenue, or collected waste volume remain relatively scarce. Approximately 23% of the reviewed literature incorporates multi-objective formulations, accounting for considerations such as workload balancing or minimizing vehicle count alongside cost reduction. Furthermore, recent studies have increasingly addressed risks to surrounding populations arising from the transport, storage, and processing of general or hazardous waste [12].

Waste collection and transportation account for over 70% of total expenses in municipal solid waste management systems, with diesel fuel consumption being a major contributor to these costs [15]. Consequently, developing cost-efficient collection strategies is essential. One of the earliest works in route optimization for solid waste collection was conducted by Beltrami and Bodin [3], who analyzed systems in New York and Washington using the classical Arc Routing Problem (ARP) approach. They employed an enhanced version of the Clarke and Wright savings algorithm [8], demonstrating significant potential for cost reduction and operational efficiency in municipal waste collection. More recently, Li et al. [15] outlined a roadmap for decision-makers in the WCVRP field, providing opportunities for more adaptive, efficient, and sustainable waste collection systems.

Numerous studies have proposed different methods to solve this problem. Genetic Algorithm (GA) and GA-based approaches are commonly employed across many works [1, 4, 7, 10, 27]. Stanković et al. [23] evaluated the effectiveness of GA, Simulated Annealing (SA), Particle Swarm Optimization (PSO), and Ant Colony Optimization (ACO) in urban waste collection, showing significant improvements in route optimization. Bouleft and Elhilali Alaoui [5] addressed a dynamic multi-compartment VRP for smart waste collection using GA. Ben-Romdhane et al. [4] used a GA-based method to optimize healthcare waste routing under waste separation policies. Quintana et al. [19] developed a VRP-based network for collecting waste vegetable oil from 49 restaurants, incorporating simultaneous pickup and delivery. Wouda et al. [29] proposed an integrated selection and routing framework using real-time data to improve route planning and service efficiency. Nurprihatin and Lestari [18] introduced a WCVRP model with multiple trips, time windows, split deliveries, heterogeneous fleets, and intermediate facilities, aiming to reduce costs and distances. Rossit and Toncovich [21] created a bi-objective model considering travel time and the visual impact of routes, promoting more community-friendly collection strategies. Additionally, many researchers have adopted hybrid approaches to solve WCVRP problems, as seen in [6, 14, 16, 22, 28, 30, 31].

This research aims to propose a practical and scalable model for local governments in Turkey. Complementing studies such as Rizvanoglu [20], which emphasize GIS-based analysis in municipal planning, this paper introduces an innovative methodology to optimize waste collection routes in Eskisehir's urban district. By combining Genetic Algorithms with Tabu Search, our study addresses the problem's complexity more effectively, aiming to produce high-quality solutions with reduced computation time. While the Genetic Algorithm enables exploration of a broad solution space, Tabu Search improves convergence by avoiding local optima and enhancing intensification and diversification. This hybrid metaheuristic approach supports the reduction of operational costs, minimization of vehicle numbers, and design of sustainable collection routes. The findings aim to offer a new perspective on urban waste management and contribute substantially to the WCVRP literature. Addressing this problem has the potential to significantly lower operational expenses while supporting environmental goals. Optimized routing leads to reduced fuel use and emissions, which in turn benefits both ecological sustainability and municipal service efficiency. Enhanced logistical planning also improves service quality and promotes more effective governance. A notable aspect of this study is also its emphasis on minimizing vehicle count, as highlighted in [25]. In both classical and environmentally extended VRP formulations, fewer vehicles are generally associated with lower emissions. However, in heterogeneous fleet VRPs, emission outcomes are highly sensitive to vehicle type. For instance, using fewer large, fuel-inefficient trucks may generate more CO_2 than using more medium-sized, efficient ones. Thus, minimizing vehicle count alone does not ensure sustainability—but it supports environmentally friendly logistics when integrated with proper vehicle selection.

The remainder of the paper is structured as follows. Section 2 presents the problem definition and the mathematical model. In Section 3 discusses the solution methods for solving waste collection VRP. Section 4 provides all computational results. Section 5 concludes the paper with final remarks.

2. Problem Definition and Mathematical Model

This section introduces the studied problem and the proposed mathematical model in detail.

2.1. **Problem definition.** The municipality is a local government authority responsible for providing administrative and public services within the boundaries of an urban district located in the province of Eskişehir, Turkey. The municipality provides various public services to city residents, including infrastructure development, environmental management, sanitation, and waste collection. Prioritizing environmentally friendly initiatives, the urban municipality aims to enhance service efficiency and protect natural resources in line with the principles of sustainable urban development. One of the municipality's core operational areas is waste management, which involves the collection and proper disposal of household waste generated in residential neighborhoods. Waste collection vehicles operate along designated routes, gathering refuse and transporting it to solid waste storage and disposal facilities. With effective planning, this system holds significant potential for cost reduction and minimization of environmental impact. The system under examination focuses specifically on the municipality's waste collection operations. In the current state, the absence of systematic route planning—along with the failure to adequately account for inter-neighborhood distances and vehicle capacities—results in decreased operational efficiency, increased costs, and greater environmental harm. The aim of this study is to develop an optimized routing system that minimizes total operational costs, fuel consumption, and the number of vehicles used. Vehicles will start from a waste collection facility, collect waste between neighborhoods until capacity is reached, and return to the same facility to complete their routes. To solve the problem, data such as neighborhood-level waste quantities, distances between neighborhoods, and fuel consumption were considered. Based on these inputs, a mathematical optimization model was developed. By addressing the inefficiencies in the current system, the proposed model seeks to enable more efficient resource use and improve the service quality.

2.2. **Mathematical model.** We present the mathematical model for the studied waste collection VRP below. The following notations are used throughout the section.

Sets

- $i, j \in N$: Set of nodes (neighborhoods and solid waste facility), $|N| = 93$
- $t \in T$: Set of vehicles, $T = \{1, 2, \ldots, k\}$

- $K \subset N$: Rural neighborhoods (visited only on specific days)
- $M \subset N$: Central neighborhoods (visited every day)
- G: Set of available days
- u_i: Continuous variable used to eliminate subtours

Parameters

- Q_t: Capacity of vehicle t (heterogeneous fleet: 25 and 27 units)
- a_i^g: Amount of waste generated by neighborhood i on day g (depends on daily intensity)
- d_{ij}: Distance between neighborhoods i and j
- M: A sufficiently large number used for constraint formulation
- w_1, w_2: Weights for the normalized objective functions
- $g^* \in G$: Day selected by the user

Decision Variables

$$x_{ijt} = \begin{cases} 1, & \text{if vehicle } t \text{ travels from node } i \text{ to node } j, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_t = \begin{cases} 1, & \text{if vehicle } t \text{ is used,} \\ 0, & \text{otherwise.} \end{cases}$$

Objective functions

The model presents a bi-objective mathematical programming approach that aims to minimize both the total distance traveled during the waste collection process and the number of vehicles used. The objective function is defined as the weighted sum of two normalized sub-objectives. The first objective, denoted as 2.1, represents the total distance traveled by the waste collection vehicles between neighborhoods. This component reflects the routing cost and is weighted by w_1. The second objective, denoted as 2.2, corresponds to the total number of vehicles used in the operation and is weighted by w_2. The weighting coefficients w_1 and w_2 determine the trade-off between minimizing travel distance and minimizing vehicle usage. For example, a higher value of w_1 prioritizes the reduction of routing costs, whereas a higher value of w_2 emphasizes the minimization of fleet size. Both components of the objective function are normalized individually. This normalization enables comparability between cost metrics of different magnitudes and provides a balanced structure for multi-criteria optimization [2.3, 2.4]. The total distance cost z_1 aggregates the distances traveled by all vehicles between all neighborhoods (see Equation 2.1). The vehicle usage indicator z_2 represents the total number of active vehicles involved in the routing (see Equation 2.2).

$$z_1 = \sum_{t \in T} \sum_{i \in N} \sum_{j \in N} d_{ij} \cdot x_{ijt} \tag{2.1}$$

$$z_2 = \sum_{t \in T} f_t \tag{2.2}$$

$$z_{1,\text{norm}} = \frac{z_1 - z_{1,\text{min}}}{z_{1,\text{max}} - z_{1,\text{min}}} \tag{2.3}$$

$$z_{2,\text{norm}} = \frac{z_2 - z_{2,\text{min}}}{z_{2,\text{max}} - z_{2,\text{min}}} \tag{2.4}$$

Under these notifications, the bi-objective mixed integer linear programming model for the waste collection vehicle routing problem is formulated as follows.

Mathematical Model

$$\text{minimize} \quad z = w_1 \cdot z_{1,\text{norm}} + w_2 \cdot z_{2,\text{norm}} \tag{2.5}$$

subject to

$$\sum_{i \in N} \sum_{j \in N} a_i^{g^*} \cdot x_{ijt} \leq Q_t \quad \forall t \in T \tag{2.6}$$

$$\sum_{i \in N} x_{ikt} - \sum_{j \in N} x_{kjt} = 0 \quad \forall k \in N \setminus \{1\},\ \forall t \in T \tag{2.7}$$

$$\sum_{\substack{i \in N \\ i \neq j}} \sum_{t \in T} x_{ijt} = 1 \quad \forall j \in N \setminus \{1\} \tag{2.8}$$

$$u_i - u_j + (n+1) \cdot \sum_{t \in T} x_{ijt} \leq n \quad \forall i, j \in N \setminus \{1\},\ i \neq j \tag{2.9}$$

$$\sum_{i \in N \setminus \{1\}} x_{i1t} = f_t \quad \forall t \in T \tag{2.10}$$

$$\sum_{\substack{i,j \in N \\ i \neq j}} x_{ijt} \leq M \cdot f_t \quad \forall t \in T \tag{2.11}$$

$$x_{ijt} = 0 \quad \forall i \in K,\ \text{if } g^* \notin \{\text{Monday}, \text{Wednesday}, \text{Friday}\} \tag{2.12}$$

$$x_{ijt} \in \{0,1\}, \quad f_t \in \{0,1\}, \quad u_i \geq 0 \tag{2.13}$$

Constraint set 2.6 ensures the capacity constraint which means that total demand of a route can not exceed the capacity of the vehicle serving this route. The flow conservation constraint 2.7 enforces that the number of vehicles entering a neighborhood equals the number of vehicles leaving it. This ensures that once a vehicle enters a neighborhood, it must also exit, preserving route continuity. To prevent redundant visits, the unique visit constraint ensures that each neighborhood is visited exactly once during the collection route. This contributes to routing efficiency and avoids unnecessary travel 2.8. The subtour elimination constraint 2.9 is incorporated to prevent the formation of smaller loops within the routes [17]. The vehicle utilization constraint 2.10 guarantees that a vehicle is considered active if it collects waste in any neighborhood, then it turns to depot. This supports accurate vehicle planning and resource tracking. Constraint 2.11 acts as a connectivity condition that establishes a relationship between the binary decision variables x and f, where M is a sufficiently large constant. When a vehicle t is employed, the constraint set (2.11) ensures that $f_t = 1$. This allows vehicle t to be included in a route, although it is not mandatory. On the other hand, if $f_t = 0$, then the left-hand side of constraint (2.11) must evaluate to zero, meaning that vehicle t cannot participate in any route. It is not necessary to include the inverse constraint, since the minimization of the first objective function—which reflects the cost associated with using vehicles—naturally drives the model toward reducing the total number of active vehicles [25]. Finally, a scheduling constraint 2.12 is introduced to limit the service of rural neighborhoods to specific days of the week. This reflects real-world operational conditions by allowing less frequent visits to rural areas, thus enhancing logistical feasibility and realism .

3. Methodology

This section provides an analysis and detailed explanation of all the methods employed in the study.

3.1. **Google OR-Tools solution method.** In this study, an algorithm based on Google OR-Tools which was first developed in 2009 by Google engineers and is specifically designed to provide optimized solutions for transportation, scheduling, and logistics problems, an open-source optimization library developed by Google, was employed to optimize urban waste collection processes. Its easy integration with Python, open-source nature, and inclusion of numerous solution strategies make it highly suitable for capacitated, multi-stop distribution problems such as urban waste collection. In this study, OR-Tools was utilized alongside a dataset constructed from inter-neighborhood distances and waste quantities. Parameters such as vehicle capacities and customer demands were defined to formulate the solution model. The problem structure was modeled using the *RoutingIndexManager* and *RoutingModel* classes, while heuristic search methods such as *Path Cheapest Arc* were employed during the solution process. The resulting routes were visualized on a map via the *OpenRouteService API* and presented to users through a digital interface. As a solution strategy, the Path Cheapest Arc algorithm was employed to generate cost-efficient (in terms of distance) routes for each vehicle. The model outputs include each vehicle's route, the total amount of waste collected, and the route completion structure. These results are presented both as textual output and as visualized maps, offering a user-friendly and digitally supported route planning system for municipal use.

The suitability of Google OR-Tools for the studied problem stems from its advantages such as high solution speed, the ability to handle real-time data, and the capacity to simultaneously minimize total distance and vehicle usage. In this context, OR-Tools was utilized to achieve faster and more efficient results on larger-scale datasets. Waste collection route optimization using Google OR-Tools is described in Algorithm 1. The visualization of vehicle routes created with OR-Tools on the map is shown in Figure 1.

3.2. **Hybrid solution method.** This section introduces the hybrid algorithm which integrates Tabu Search (TS) with Genetic Algorithm (GA). TS is a single-solution metaheuristic that leverages memory structures to navigate escape from local optima, initially introduced by Glover [11], has since been widely applied to a variety of combinatorial optimization problems including different types of vehicle routing problems [25]. To prevent cycling back to previously explored solutions, the algorithm incorporates memory-based structures—commonly referred to as the tabu list—which store key attributes of recent solutions. It has been effectively adapted to the Waste Collection VRP (WCVRP), particularly under constraints such as time windows and vehicle capacity limits. Additionally, TS has been employed in bi-objective urban waste collection models to minimize both routing distance and environmental emissions.

GA is a population-based metaheuristic inspired by the concept of natural selection [13], where the fittest individuals are more likely to survive and reproduce. It evolves a set of candidate solutions over successive generations to progressively approximate high-quality outcomes. GA is particularly well-suited for addressing complex optimization problems such as vehicle routing and scheduling [24] that are typically intractable using exact methods. GAs operate on populations of candidate solutions, optimizing them via iterative crossover, mutation, and selection processes. The stochastic nature of GAs enables them to handle the complexity inherent in urban waste routing problems. While direct GA for WCVRP is less prevalent, hybrid GA-TS approaches demonstrate GA's utility in producing strong initial populations and facilitating multi-trip route considerations. Hybrid metaheuristic approaches combine the strengths of multiple algorithms to address the limitations of individual methods. The integration of TS and GA has emerged as a particularly powerful strategy for solving complex optimization problems—especially those that are combinatorial, nonlinear, and high-dimensional.

Here are some important features of the hybrid algorithm (HA) used in this study:

GA excels at exploration: It works on a population of candidate solutions, encouraging diversity and global exploration of the solution space. Through mechanisms like crossover and mutation, GA avoids

Algorithm 1 Waste Collection Route Optimization using Google OR-Tools

Input: Day of operation, number of small and large vehicles
Output: Optimized vehicle routes, total distance (z_1), number of vehicles used (z_2)
Step 1: User Input and Initialization
Prompt user for inputs; define vehicle capacities and parameters
Step 2: Data Importation
Load distance matrix, waste generation data, and coordinates from Excel
Step 3: Data Preprocessing
Normalize neighborhood names; apply weekday-based scaling to waste data
Step 4: Feasibility Check
Calculate total waste and vehicle capacity
if waste $>$ capacity **then**
    Prompt user to adjust vehicle counts
end if
Step 5: Demand Construction
Identify nodes and calculate waste demand per neighborhood
Step 6: Distance Matrix Generation
Build a square matrix for active nodes
Step 7: Routing Model Setup
Initialize OR-Tools: `RoutingIndexManager`, `RoutingModel`
Define cost function and capacity constraints
Step 8: Optimization
Apply `PathCheapestArc` strategy
Set time limit (120 seconds)
Step 9: Solution Handling
if solution exists **then**
    Generate vehicle routes
    Calculate distance and waste per route
    Print structured output
else
    Report infeasibility to user
end if
Step 10: Visualization
Use Folium to visualize routes with distinct colors
Step 11: Objective Evaluation
Compute z_1: total distance traveled
Compute z_2: number of vehicles used

premature convergence and is effective in discovering promising regions of the search space. TS excels at exploration: It is a single-solution local search method that refines candidate solutions by intensively searching their neighborhoods. Its memory-based "tabu list" prevents cycling and allows controlled deterioration, enabling the algorithm to escape local optima. By combining them, GA provides broad exploration and maintains solution diversity and TS offers deep exploitation to fine-tune and locally optimize GA-generated candidates. The general steps of HA algorithm is given below.

- Step1. Initialization of the Population: A diverse set of initial candidate solutions (individuals) is generated randomly or based on heuristics. These individuals represent potential routing plans and form the initial population for the genetic algorithm.

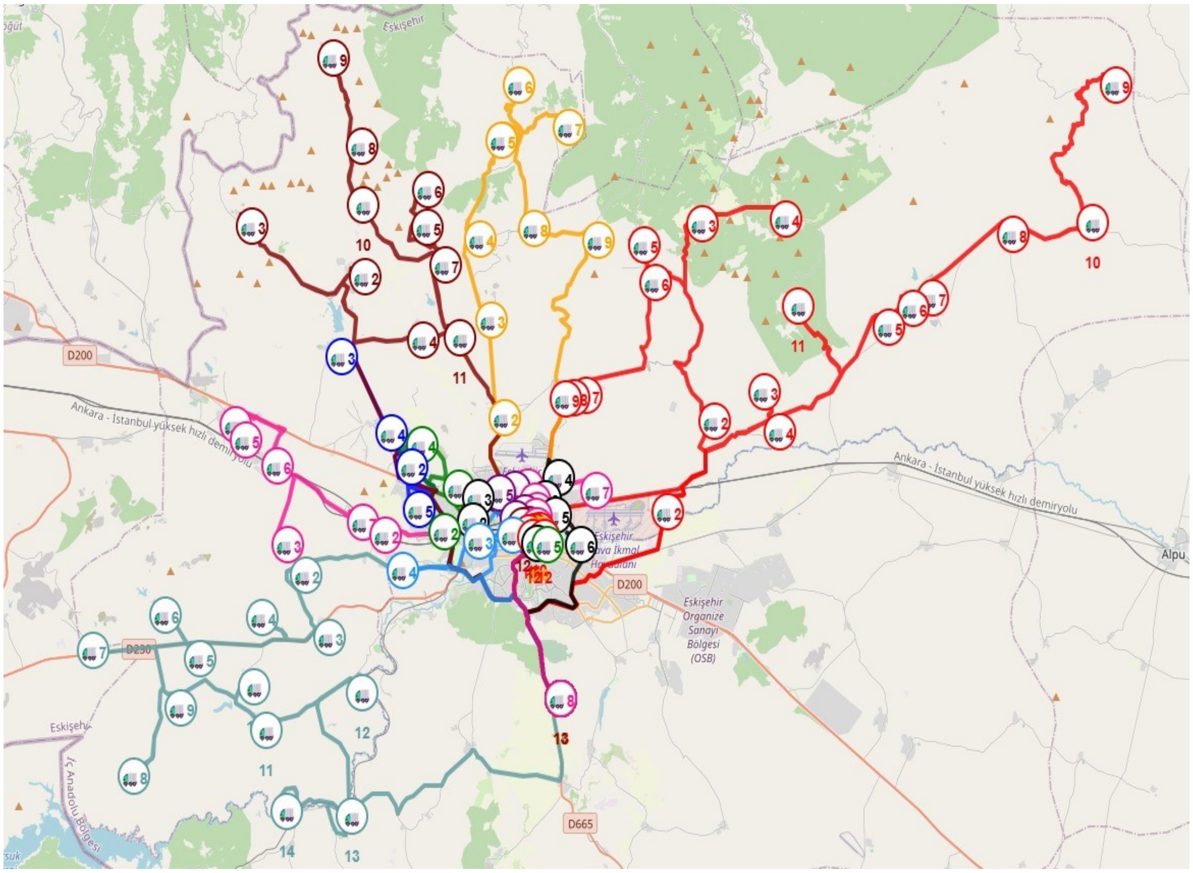

Figure 1. Visualization of vehicle routes created with OR-Tools on the map

- Step 2. Crossover and Mutation Operations in GA: Genetic operators are applied to evolve the population. Crossover combines parts of two parent solutions to produce offspring, aiming to inherit the most promising traits. Mutation introduces small random changes to individuals to maintain diversity and avoid premature convergence.
- Step 3. Improvement via Tabu Search: Each individual solution is further refined using Tabu Search, a local search method that explores the neighborhood of the current solution while avoiding cycles and previously visited (tabu) solutions. This step enhances the local optimality of the population.
- Step 4. Selection of the Best Solution: Among the final set of evolved and locally optimized solutions, the one with the best performance according to the predefined objective function (e.g., minimal total distance, minimal number of vehicles) is selected as the optimal solution.

4. Computational Results

This section introducesthe data collection and analysis; the results for the toy problem and the results for the big sized real data used in this study.

4.1. **Data collection and analysis.** Initially, data were collected from the municipal authority pertaining to the designated neighborhood within the study area including fuel consumption, waste tonnage, and number of containers over a period of five days. These data were processed by calculating the average waste tonnage to ensure consistency in the analysis .In the first phase of the study, data obtained from designated neighborhood were used as a reference. Using the Python programming language, the waste tonnage and estimated number of containers for the remaining 91 neighborhoods (1 waste collection point-depot and 92 neighborhoods) were calculated through a proportional estimation method. The variables required for this estimation—such as the number of streets and the total circulation length within each neighborhood—were obtained using QGIS in Figure 2, a Geographic Information System (GIS) platform.

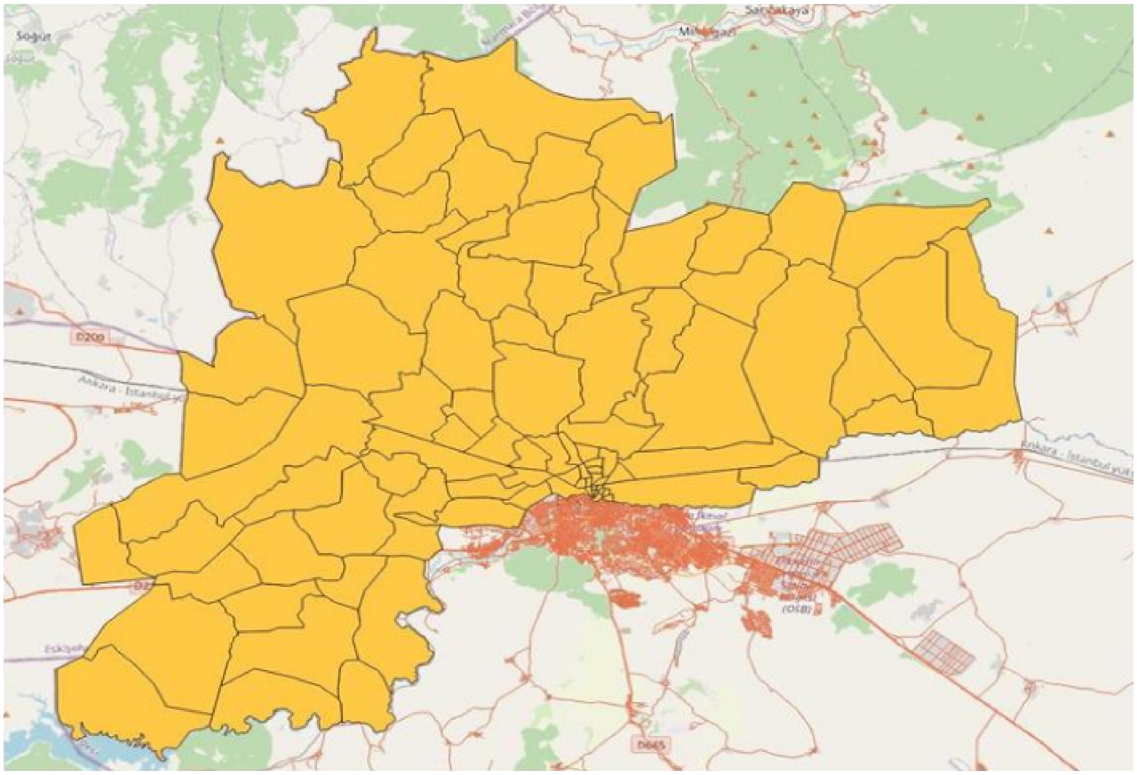

FIGURE 2. Visualization of the urban district via QGIS application

The road data used in this study were derived from a dataset classifying the road network. In this dataset, roads are categorized under the "fclass" column. However, due to the physical limitations of waste collection vehicles, certain types of roads were excluded from the analysis. Only roads deemed accessible to waste collection vehicles were filtered and considered in the evaluation. For instance, road types such as footway, steps, pedestrian, and living street, which are not suitable for vehicle access, were removed from the dataset. Following this filtering process, road classes appropriate for vehicle use—such as residential, secondary, and primary roads—were retained for analysis. This step ensured that the road network included only operationally usable segments, allowing for the generation of more accurate and realistic optimized routes. A linear regression model was developed using both the available data and values obtained from the implementation. The estimations and resulting outputs are presented below. As shown in Algorithm 2, the known number of waste containers and streets in the studied neighborhood were used as the basis for estimating the number of waste containers in other neighborhoods. First, the number of streets in each neighborhood was determined using QGIS, a Geographic Information System (GIS). Then, the ratio between the number of streets and waste containers was calculated. This ratio was applied to the number of streets in other neighborhoods to estimate their respective container counts.

In Algorithm 3 below, the known number of waste containers and the average weekly total waste tonnage for the studied neighborhood were used as the basis for estimating the waste tonnage in other neighborhoods. First, the average amount of waste per container was calculated using data from the neighborhood. This value was then multiplied by the estimated number of containers in each neighborhood

Algorithm 2 Estimation of Container Counts Based on Street Data

Input: Known number of streets and containers for reference neighborhood
Input: Excel file containing number of streets per neighborhood
Output: Excel file with estimated number of containers per neighborhood
Set reference values:
  $\texttt{known street ccount} \leftarrow 233$
  $\texttt{known container count} \leftarrow 309$
Read Excel file containing street counts per neighborhood
Rename columns to: `[Neighborhood, Street Count]`
For each neighborhood:
  Estimate container count:
    $\texttt{estimated containers} \leftarrow \left(\frac{\texttt{street\_count}}{\texttt{known street count}}\right) \times \texttt{known container count}$
Round all estimates to the nearest integer
Save results to Excel file
Print confirmation message: ″Excel file saved″

to derive the predicted waste tonnage. The results were compiled into a table containing neighborhood names, street counts, and estimated waste container numbers, and were subsequently saved as an Excel file. Using QGIS software, the distances between 92 neighborhoods in the urban district of Eskisehir and a single solid waste disposal point were calculated and prepared for use in the analysis.

Algorithm 3 Estimation of Neighborhood Waste Tonnage Based on Container Counts

Input: Reference data from the studied neighborhood ($\texttt{container count}, \texttt{total tonnage}$)
Input: Excel file with estimated container counts for other neighborhoods
Output: Excel file with estimated waste tonnage
Define reference data
  container count $\leftarrow 309$
  total tonnage $\leftarrow 11.560$
Compute waste per container:
  waste per container $\leftarrow \frac{\text{total tonnage}}{\text{container count}}$
Read Excel file containing estimated container counts for neighborhoods
For each neighborhood:
  Compute estimated waste tonnage:
    estimated tonnage $\leftarrow$ estimated containers $\times$ waste per container
Round all estimates to two decimal places
Save results to Excel file
Print confirmation message: "Excel file saved"

4.2. **Computational results for the toy problem.** A series of computational experiments were conducted to verify the efficiency of the proposed solution method. The numerical experiments have been carried out on a PC with a Casper Excalibur G870 with an Intel Core i5-13420H processor and 32 GB RAM. We used GAMS software-version 48- to solve the toy problem with the given mathematical model optimally. Table 1 and Table 2 present the data used for toy problem; Table 3 shows the obtained results.

From Table 3, the detailed results for the mathematical model is analyzed. If $w_1 = 0.5$ and $w_2 = 0.5$, the corresponding result, $z_1 = 112.8451$ and $z_2 = 4$ which has found optimally. This configuration reflects a balanced compromise between minimizing distance and minimizing vehicle usage. Four vehicles are used,

TABLE 1. Model Parameters and Neighborhood Waste Amounts

Parameter	**Value**
Vehicle capacity (Q)	25
$z_1^{\min}$	2
$z_1^{\max}$	200
$z_2^{\min}$	2
$z_2^{\max}$	4

Neighborhood (i)	**Waste Amount (tons)**
1	0.00
2	20.43
3	11.56
4	2.24
5	7.78
6	7.26
7	20.73
8	3.14

TABLE 2. Distance Matrix Between Neighborhoods (km)

From $\setminus$ To	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0.0000	16.6976	12.4516	10.0936	11.8034	12.6836	14.5837	11.4286
2	15.9789	0.0000	6.5475	8.7393	7.5370	9.5497	5.7301	8.2781
3	13.1137	6.7345	0.0000	4.0968	2.2527	5.0928	3.6752	3.6356
4	9.5891	8.8528	3.8234	0.0000	1.6879	2.8629	4.4682	1.6079
5	10.6617	7.9601	2.2527	2.6442	0.0000	3.6403	3.5755	2.1830
6	12.5388	9.2627	5.0083	2.8197	3.5557	0.0000	4.8781	1.6595
7	15.8670	5.7586	3.9198	5.0502	3.8480	3.9736	0.0000	4.5890
8	11.2367	8.6650	3.6356	1.5176	2.1830	1.6586	4.2804	0.0000

TABLE 3. GAMS results for the toy problem

w_1	w_2	z_1	z_2	Vehicle routes	Vehicles used
0.5	0.5	112.8451	4	1-2-1	vehicle 5
				1-3-5-1	vehicle 7
				1-7-4-1	vehicle 2
				1-6-8-1	vehicle 3
0.9	0.1	112.8451	4	1-2-1	vehicle 5
				1-3-5-1	vehicle 7
				1-7-4-1	vehicle 2
				1-6-8-1	vehicle 3
0.1	0.9	170.60	2	1-4-7-8-1	vehicle 1
				1-3-5-6-2-1	vehicle 2

and the distance is relatively high. This is a classic trade-off solution in multi-objective settings. If $w_1 = 0.9$ and $w_2 = 0.1$, the result is the same. Despite prioritizing distance minimization, the solution remains unchanged. This suggests that the same routing configuration remains Pareto optimal even when the weight for distance increases significantly. It implies a robust solution across multiple preference structures. But If $w_1 = 0.1$ and $w_2 = 0.9$,, which means that the number of vehicles used is 9 times important than the first objective the result changes and 2 vehicle are used. Prioritizing fewer vehicles leads to route consolidation. Total route cost increases by over 50% (from 112.85 to 170.60), but the number of vehicles used is halved. This illustrates the trade-off between fleet minimization and operational cost. Routes become longer and more complex, potentially increasing driver workload and service time. Scenario 3 significantly reduces vehicle number at the expense of total distance. This reflects a practical decision-making axis between operational efficiency (cost/distance) and resource minimization (fleet size). For cost-sensitive operations (e.g., fuel-dominated logistics), Scenario 1/2 is preferable. For labor or vehicle-limited operations (e.g., limited fleet availability), Scenario 3 offers a viable alternative despite higher operational cost.

4.3. **Computational results for the case study.** In this study, a mathematical vehicle routing model was developed to enhance the efficiency, environmental sustainability, and digital manageability of urban waste collection operations. The model was solved using both deterministic linear optimization via Google OR-Tools and metaheuristic optimization (through a Genetic Algorithm combined with Tabu Search). Scenario-based comparisons conducted across different dates revealed that Google OR-Tools generally produced shorter routes and lower objective function values. However, it was also observed that the hybrid approach achieved solutions with fewer vehicles in certain scenarios. The below subsections 4.4 and 4.5 include the results of OR-tools and hybrid algorithm for the studied data. All computational results are explained in detail.

4.4. **Computational results of OR-tools.** Due to GAMS's inability to solve the large-scale real-world problem within a reasonable timeframe, the problem was addressed using both Google OR-Tools and the proposed hybrid algorithm. In this study, a solution model was developed using the Google OR-Tools library in Python to optimize the routing of waste collection vehicles. The model is based on the Capacitated Vehicle Routing Problem (CVRP) framework and aims to ensure that waste collection tasks between neighborhoods are carried out in the most efficient manner, subject to vehicle capacity constraints. The code structure allows users to input the day of operation along with the number of small and large vehicles, enabling the system to model daily operational needs. Waste quantities are dynamically determined based on these inputs. Neighborhood coordinates and distance matrices, obtained through QGIS, are directly integrated into the model. The routing structure is built using OR-Tools' RoutingIndexManager and RoutingModel classes, while capacity constraints are defined through appropriate functions. The Path Cheapest Arc algorithm is employed as the solution strategy to generate the most cost-efficient (i.e., shortest distance) routes for each vehicle. The model outputs include details such as each vehicle's route, the amount of waste collected, and how the route is completed. These outputs are provided both in textual format and as visualizations on a map, offering a user-friendly and digitally supported route planning system tailored for municipal use. Waste collection route optimization using Google OR-Tools is described in Algorithm 4.

Table 4 outlines the optimized waste collection plan for Monday within the urban district, incorporating both small and large vehicles. The model schedules 13 vehicles in total—significantly fewer than the available fleet of 30 (20 small, 10 large)—highlighting efficient vehicle utilization. Each vehicle route begins and ends at a centralized waste collection point, servicing a series of neighborhoods in a closed-loop pattern. The collected waste per route ranges between approximately 23.81 kg and 26.95 kg, indicating effective load balancing across vehicles and adherence to vehicle capacity constraints. The optimization

Algorithm 4 Waste Collection Route Optimization using Google OR-Tools

```
Input: Day of operation, number of small and large vehicles
Output: Optimized vehicle routes, total distance (z1), number of vehicles used (z2)
User Input and Initialization
Prompt user for inputs; define vehicle capacities and parameters
Data Importation
Load distance matrix, waste generation data, and coordinates from Excel
Data Preprocessing
Normalize neighborhood names; apply weekday-based scaling to waste data
Feasibility Check
Calculate total waste and vehicle capacity
if waste > capacity then
    Prompt user to adjust vehicle counts
end if
Demand Construction
Identify nodes and calculate waste demand per neighborhood
Distance Matrix Generation
Build a square matrix for active nodes
Routing Model Setup
Initialize OR-Tools: RoutingIndexManager, RoutingModel
Define cost function and capacity constraints
Optimization
Apply PathCheapestArc strategy
Set time limit (120 seconds)
Solution Handling
if solution exists then
    Generate vehicle routes
    Calculate distance and waste per route
    Print structured output
else
    Report infeasibility to user
end if
Visualization
Use Folium to visualize routes with distinct colors
Objective Evaluation
Compute z1: total distance traveled
Compute z2: number of vehicles used
```

objective focused on minimizing both the total route distance (z_1) and the number of vehicles used (z_2), with a weighted composite function yielding a normalized objective value (z) of 0.0402. The total travel distance across all routes was 869.33 km, and the solution achieved coverage of the entire service area with only 13 vehicles, implying a 56.7% reduction in active fleet size compared to maximum availability. The solution contributes to operational efficiency by: reducing total fuel consumption through route minimization; limiting carbon emissions via constrained fleet deployment; maintaining full area coverage and collection service standards. This result confirms the model's effectiveness in addressing the WCVRP under real-world constraints. It demonstrates the potential of heuristic-based routing algorithms to significantly optimize urban waste management processes while aligning with environmental sustainability goals.

Table 4. Waste Collection Routes and Objective Function Values – Monday

Vehicle	Route	Total Waste (kg)
Small Vehicle 1	Waste Collection Point → Cumhuriyet → Yarımca → Tandır → Bozdağ → Karadere → Muttalip Emirler → Muttalip Orta → Hacıseyit → Hayriye → Hacı Ali Bey → Cumhuriyet → Waste Collection Point	24.74
K118	Waste Collection Point → Uludere → Sırintepe → Yeni → Waste Collection Point	24.70
K119	Waste Collection Point → Karagözler → Turgutlar → Hisar → Çukurhisar Yeni → Satılmışoğlu → Boyacıoğlu → Waste Collection Point	23.81
8201	Waste Collection Point → Yusuflar → Yeni Akçayır → Yörük Akçayır → Aşağı Kartal → Yukarı Kartal → Nemli → Çanakıran → Mollaoğlu → Tokmak → Musaözü → Kızılınler → Gökçekısık → Yeni İncesu → Waste Collection Point	24.70
8202	Waste Collection Point → Hasanbey → Ahiler → Gökdere → Kızılcaören → Yakaşan → Gündüzler → Beyazaltın → Yaylaca → Waste Collection Point	26.84
8203	Waste Collection Point → Aşağısöğütönü → Keskin → Yukarısöğütönü → Batıkent → Waste Collection Point	26.21
8204	Waste Collection Point → Çamlıca → Zincirlikuyu → Yaşamkent → Yeni → Waste Collection Point	26.95
8205	Waste Collection Point → Güllük → Bahçelievler → Sütlüce → Yeşiltepe → Yenibağlar → Waste Collection Point	26.87
8206	Waste Collection Point → Sakintepe → Kozkayı → Sulukaraağaç → Tekeciler → Atalan → Atalantekke → Bektaşpınar → Hekimdağ → Mamure → İhsaniye → Waste Collection Point	26.91
8207	Waste Collection Point → Eğriöz → Uludere → Alınca → Buldukpınarı → Avlamış → Kavacık → Behçetiye → Çalkara → Karaçobanpınarı → Emirceoğlu → Eskibağlar → Waste Collection Point	26.07
8208	Waste Collection Point → Hoşnudiye → Ertuğrulgazi → Sazova → Waste Collection Point	25.14
8209	Waste Collection Point → Mustafa Kemal Paşa → Zafer → Gazipaşa → Şarhöyük → Şeker → Waste Collection Point	26.29
8210	Waste Collection Point → Ömerağa → Tunalı → Fatih → Kumlubel → Esentepe → Fevziçakmak → Waste Collection Point	26.27
	Total Distance (z_1)	869.33 km
	Number of Vehicles Used (z_2)	13
	Normalized Objective Function (z)	0.0402

4.5. **Computational results of hybrid algorithm.** Hybrid algorithm is implemented by using Python in Google Colab. Hybrid algorithm is explained in detail in Algorithm 5.

Based on the user-provided inputs such as the day of operation and number of vehicles, daily waste generation is dynamically calculated. Using inter-neighborhood distance data and waste quantities obtained from QGIS, a distance matrix and a demand vector are constructed. The code then determines the sequence of neighborhood visits for each vehicle, penalizing capacity violations to ensure the feasibility of the model. At the end of the solution process, the route for each vehicle, the amount of waste collected, and the vehicle type (small/large) are printed to the console, providing operationally actionable outputs. Additionally, a normalized objective function value is computed as a multi-objective performance metric. The data used for an example of the case study is given in Table 5. The obtained solution in Table 6 analyzes the result of a daily waste collection report for Monday.

Table 6 presents the outcome of a hybrid optimization algorithm applied to the Vehicle Routing Problem (VRP) for municipal solid waste collection in the urban district. The solution encompasses 13 routes serviced by a mix of 8 large and 5 small vehicles, each route beginning and ending at the central solid waste facility. The fleet is composed of heterogeneous vehicles, categorized as large and small, to match neighborhood accessibility and expected waste volumes. The hybrid algorithm allocates 8 large vehicles

Algorithm 5 Hybrid Algorithm

Require: Distance matrix, demands, vehicle capacities, population size P, generations G, mutation rate μ, crossover rate ρ, tabu tenure L, tabu iterations T
Ensure: Feasible solution $best$ with minimized cost

```
Initialize population P with feasible solutions using create_individual()
Evaluate fitness of each individual in P using calc_fitness()
for gen = 1 to G do
    Sort population P by fitness
    Select elite individuals to carry to next generation
    while new population size < |P| do
        Select two parents from elite pool
        if rand() < ρ then
            child ← crossover(parent_1, parent_2)
        else
            child ← mutate(parent_1)
        end if
        if rand() < μ then
            child ← mutate(child)
        end if
        child ← tabu_search(child, L, T)
        Add child to new population
    end while
    P ← new population
end for
return best individual from P
```

TABLE 5. Daily Waste Collection Summary (Monday)

Parameter	Value
Number of Small Vehicles	10
Number of Large Vehicles	10
Total Distance Traveled	1985.14 km
Total Vehicles Used	13
Normalized Objective Function Value	0.0431

to longer and denser routes, typically servicing extended rural or semi-urban neighborhoods with higher cumulative waste volumes. 5 small vehicles to shorter and more compact urban routes, likely where maneuverability is critical. This heterogeneous deployment suggests that the model effectively incorporates vehicle capacities and route feasibility into its optimization logic, enhancing real-world applicability. The amount of waste collected per vehicle ranges from 23.15 kg (Small 5) to 26.97 kg (Large 2), closely aligning with presumed vehicle capacity constraints. The narrow variance in load values indicates excellent load balancing and an efficient distribution of waste collection responsibilities across the fleet. It minimizes the risk of overloading while maximizing capacity utilization, which contributes directly to operational stability. The routes vary in complexity, with large vehicles (e.g., Large 4 and Large 7) covering up to 15+ distinct collection points, while small vehicles typically serve 3–6 neighborhoods per trip. This differentiation reflects the algorithm's capacity to handle varying degrees of spatial dispersion and adapt route length to vehicle capabilities. The inclusion of both high-density urban centers and remote suburban areas within

TABLE 6. Hybrid Algorithm Results for Vehicle Routes and Collected Waste (Monday)

Vehicle	Route	Waste (kg)
Large 1	Solid Waste Facility → Zafer → Tandır → Kavacık → Yaylacık → Şeker → Karagözler → Aşağı Kartal → Solid Waste Facility	26.80
Small 1	Solid Waste Facility → Yeşiltepe → Zincirlikuyu → Eğriöz → Karaçobanpınarı → Solid Waste Facility	24.43
Large 2	Solid Waste Facility → Emirceoğlu → Uludere → Hisar → Satılmışoğlu → Yörük Akçayır → Behçetiye → Sulukarağaç → Hayriye → Solid Waste Facility	26.97
Large 3	Solid Waste Facility → Yusuflar → Ömerağa → Kumlubel → Muttalıp Emirler → Bozdağ → Aşağısöğütönü → Boyacıoğlu → Turgutlar → Yukarı Kartal → Musaözü → Gökdere → Solid Waste Facility	26.72
Large 4	Solid Waste Facility → Beyazaltın → Kumlubel → Çalkara → Keskin → Yaşamkent → Tekeciler → Buldukpınarı → Çanakkıran → Cumhuriyet → İhsaniye → Hacı Ali Bey → Nemli → Hacıseyit → Yeni → Solid Waste Facility	26.41
Small 2	Solid Waste Facility → Uluönder → Şirintepe → Solid Waste Facility	24.70
Small 3	Solid Waste Facility → Batıkent → Esentepe → Avlamış → Yarımca → Solid Waste Facility	24.88
Small 4	Solid Waste Facility → Yeni Akçayır → Sakintepe → Sütlüce → Mustafa Kemal Paşa → Taycılar → Solid Waste Facility	24.55
Large 5	Solid Waste Facility → Fevziçakmak → Fatih → Bektaşpınar → Cumhuriyet → Hasanbey → Kızılcören → Danişment → Ahiler → Solid Waste Facility	26.92
Large 6	Solid Waste Facility → Hoşnudiye → Gazipaşa → Muttalıp Koyunlar → Şarhöyük → Eskibağlar → Mollaoğlu → Çukurhisar Yeni → Laradere → Solid Waste Facility	26.82
Large 7	Solid Waste Facility → Tunalı → Muttalıp Orta → Atalantekke → Güllük → Yenibağlar → Yukarı Söğütönü → Yeni İncesu → Mamure → Işıklar → Hekimdağ → Atalan → Kızılinler → Gökçekısık → Solid Waste Facility	26.94
Large 8	Solid Waste Facility → Çamlıca → Alınca → Gündüzler → Solid Waste Facility	26.19
Small 5	Solid Waste Facility → Takmak → Sazova → Ertuğrulgazi → Bahçelievler → Solid Waste Facility	23.15

optimized routes indicates strong spatial adaptability of the model. The detailed vehicle routing structure suggests that the hybrid algorithm—likely combining heuristics such as Genetic Algorithms and Tabu Search—successfully navigates a complex solution space with multi-dimensional constraints (capacity, distance, route length, vehicle type). The consistency of near-capacity loads and strategic route segmentation indicate the algorithm's competence in balancing exploration (diversification) and exploitation (intensification) phases. By considering the operational and environmental benefits it is observed that fewer vehicles used: only 13 out of a larger potential fleet (assumed from prior context) were utilized. Optimized routes reduce unnecessary travel, minimizing fuel use and emissions. All designated neighborhoods were included in the plan without redundancy.

Figure 3 below provides a comparative analysis of both methods across three performance indicators: total distance, number of vehicles used, and the aggregated objective function value. A review of this figure shows that Google OR-Tools consistently produced lower distances for almost all scenarios, indicating its effectiveness in distance-based optimization. In terms of vehicle usage, both methods yielded similar results, though the hybrid algorithm occasionally achieved solutions with fewer vehicles, demonstrating its flexibility and capability for generating alternative solutions. Regarding the overall performance metric, OR-Tools generally outperformed the hybrid method by producing lower values. Beyond generating optimal solutions, the study also developed a web-based user interface to facilitate the practical application of the model. This interface enables municipal personnel to easily and quickly select neighborhoods during daily planning and visualize the most efficient routes. Based on user input, the system provides route suggestions for both small and large-capacity vehicles, and displays them on a map using the *OpenRouteService API*. This framework offers a user-friendly digital solution and creates a practical platform that decision-makers can readily implement in real-world operations.

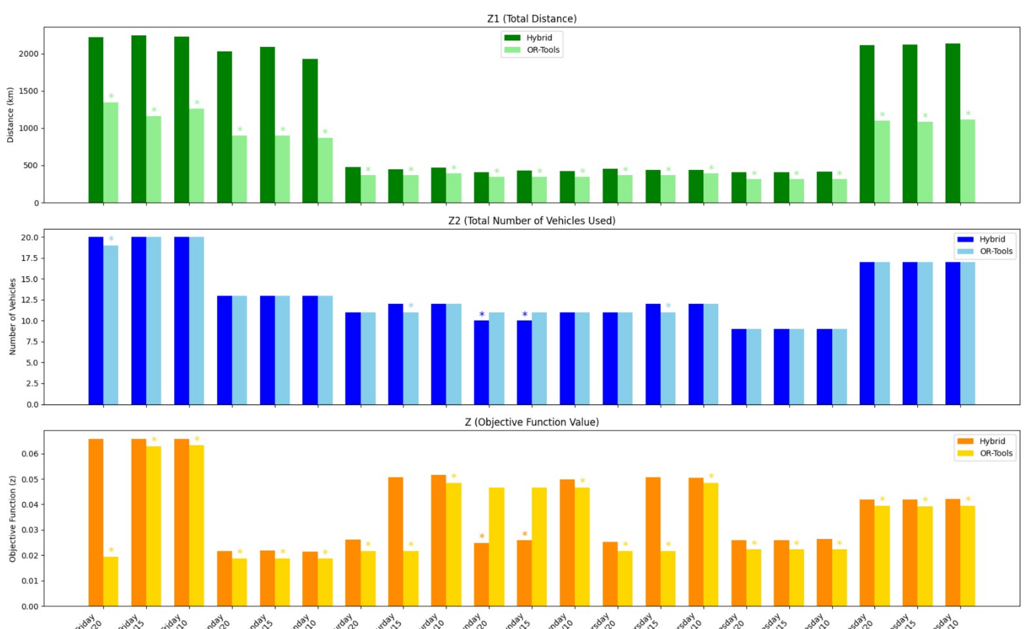

Figure 3. Comparison of OR-Tools and the Hybrid Method

5. Conclusion

This study focuses on enhancing the efficiency of waste collection operations in the urban district of Eskisehir, Turkey. Driven by the challenges of growing urbanization and population density, the research addresses the need for more effective and sustainable municipal waste management. A region-specific solution to the Vehicle Routing Problem is developed using input data such as neighborhood-level waste generation, container utilization rates, and vehicle capacity constraints. The proposed model aims to optimize operational performance by minimizing both overall costs and the number of vehicles required. In line with environmental sustainability goals, the study offers a practical and scalable tool for local administrations. A hybrid metaheuristic approach—combining Genetic Algorithm and Tabu Search, both well-established in the literature—is employed. The performance of the hybrid method is assessed through computational experiments on a real-world case, with comparative analysis against Google OR-Tools. The findings aim to provide a novel contribution to the field of urban waste management and enrich the academic discourse. Google OR-Tools method can be adopted as a primary tool for daily operational planning in municipalities due to its ability to deliver stable and low-cost solutions within a short time frame. The hybrid algorithm can serve as a valuable tool for generating alternative solutions under varying time frames or seasonal fluctuations. As a future work, the proposed model can be further expanded in the future by incorporating carbon emissions, labor costs, and traffic congestion, making it more comprehensive. The developed model and digital interface are not limited to the urban district alone; they can be easily implemented by other local governments with similar infrastructure. The proposed algorithm can be also applied to other combinatorial optimization problems, by considering other objectives. Moreover, incorporating machine learning methods to dynamically adjust parameters during execution may enhance the system's adaptability and overall efficiency across different problem instances.

Statements and Declarations

The authors declare that they have no conflict of interest, and the manuscript has no associated data.

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References

[1] S. Z. Aliahmadi, F. Barzinpour, and M. S. Pishvaee. A fuzzy optimization approach to the capacitated node-routing problem for municipal solid waste collection with multiple tours: A case study. *Waste Management and Research*, 38(3):279-290, 2020.

[2] M. Alpaslan Takan and R. Kasimbeyli. A hybrid subgradient method for solving the capacitated vehicle routing problem. *Journal of Nonlinear and Convex Analysis*, 21(2):413-423, 2020.

[3] E. J. Beltrami and L. D. Bodin. Networks and vehicle routing for municipal waste collection. *Networks*, 4(1):65–94, 1974.

[4] H. Ben-Romdhane, N. Ouertani, S. Krichen, and I. Nouaouri. On optimizing healthcare waste routing systems using waste separation policies: A case study. *Applied Soft Computing*, 146:Article ID 110615, 2023.

[5] Y. Bouleft and A. Elhilali Alaoui. Dynamic multi-compartment vehicle routing problem for smart waste collection. *Applied System Innovation*, 6(1):Article ID 30, 2023.

[6] C. Cao, J. Li, J. Liu, J. Liu, H. Qiu, and J. Zhen. Sustainable development-oriented location-transportation integrated optimization problem regarding multi-period multi-type disaster medical waste during COVID-19 pandemic. *Annals of Operations Research*, 335(3):1401-1447, 2024.

[7] C. Cheng, R. Zhu, A. M. Costa, R. G. Thompson, and X. Huang. Multi-period two-echelon location routing problem for disaster waste clean-up. *Transportmetrica A: Transport Science*, 18(3):1053-1083, 2022.

[8] G. Clarke, and J. W. Wright. Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, 12(4):568-581, 1964.

[9] G. B. Dantzig and J. H. Ramser. The Truck Dispatching Problem. *Management Science*, 6:80-91, 1959.

[10] N. Elshaboury and M. Marzouk. Optimizing construction and demolition waste transportation for sustainable construction projects. *Engineering, Construction and Architectural Management*, 28(9):2411-2425, 2021.

[11] F. Glover. Future paths for integer programming and links to artificial intelligence. *Computers and Operations Research*, 13:533–549, 1986.

[12] K. Govindan, H. Soleimani, and D. Kannan. Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research*, 240(3):603–626, 2015.
[13] J. H. Holland. *Adaptation in Natural and Artifcial Systems: An Introductory Analysis with Applications to Biology, Control, and Artifcial Intelligence*. University of Michigan Press, USA, 1975.
[14] D. Jorge, A. P. Antunes, T. R. P. Ramos, and A. P. Barbosa-Póvoa. A hybrid metaheuristic for smart waste collection problems with workload concerns. *Computers and Operations Research*, 137:Article ID 105518, 2022.
[15] W. Li, T. F. Ng, H. Ibrahim, and S. L. Wang. A literature review of the state of the art of sustainable waste collection and vehicle routing problem. *Journal of the Air and Waste Management Association*, 75(1):3-26, 2025.
[16] S. Kaya. A hybrid firefly and particle swarm optimization algorithm with local search for the problem of municipal solid waste collection: a real-life example. *Neural Computing and Applications*, 35(9):7107-7124, 2023.
[17] C. E. Miller, A. W. Tucker, and R. A. Zemlin. Integer programming formulations and traveling salesman problems. *Journal of the Association for Computing Machinery*, 7:326–329, 1960.
[18] F. Nurprihatin and A. Lestari. Waste collection vehicle routing problem model with multiple trips, time windows, split delivery, heterogeneous fleet and intermediate facility. *Engineering Journal*, 24(5):55-64, 2020.
[19] L. Quintana, Y. Herrera-Mena, J. L. Martínez-Flores, M. A. Coronado, G. Montero, and P. Cano-Olivos. Design of waste vegetable oil collection networks applying vehicle routing problem and simultaneous pickup and delivery. *Acta Logistica*, 7(4):261-268, 2020.
[20] O. Rizvanoglu. Kati Atik Toplama Guzergah Optimizasyonu: Haliliye (Sanliurfa) Ilcesi Ornegi. Doctoral Dissertation, 2018.
[21] D. Rossit and A. Toncovich. Bi-objective optimization of a VRP problem applied to urban solid waste collection through a model that includes the visual attraction of routes. ArXiv:2405.00068, 2024.
[22] C. Shang, L. Ma, Y. Liu, and S. Sun. The sorted-waste capacitated location routing problem with queuing time: A cross-entropy and simulated-annealing-based hyper-heuristic algorithm. *Expert Systems with Applications*, 201:Article ID 117077, 2022.
[23] A. Stankovic, D. Markovic, G. Petrovic, and Z. Cojbasic. Metaheuristics for the waste collection vehicle routing problem in urban areas. *Facta Universitatis, Series: Working and Living Environmental Protection*, 17(1):1-16, 2020.
[24] M. A. Takan. A novel MIP model and a hybrid genetic algorithm for operation outsourcing in production scheduling with carbon tax policy. *Expert Systems with Applications*, 251:Article ID 123983, 2024.
[25] M. A. Takan and R. Kasimbeyli. Multiobjective mathematical models and solution approaches for heterogeneous fixed fleet vehicle routing problems. *Journal of Industrial and Management Optimization*, 17(4):2073-2095, 2021.
[26] E. G. Talbi. *Metaheuristics: From Design to Implementation*, John Wiley and Sons, 2009.
[27] H. Wang, W. Yi, and Y. Liu. Optimal route design for construction waste transportation systems: Mathematical models and solution algorithms. *Mathematics*, 10(22):Article ID 4340, 2022.
[28] Z. Wei, C. Liang, and H. Tang. A Cross-Regional Scheduling Strategy of Waste Collection and Transportation Based on an Improved Hierarchical Agglomerative Clustering Algorithm. *Computational Intelligence and Neuroscience*, 2022:Article ID 7412611, 2022.
[29] N. A. Wouda, M. Aerts-Veenstra, and N. van Foreest. An integrated selection and routing policy for urban waste collection. ArXiv:2409.13386, 2024.
[30] M. Yazdani, K. Kabirifar, B. E. Frimpong, M. Shariati, M. Mirmozaffari, and A. Boskabadi. Improving construction and demolition waste collection service in an urban area using a simheuristic approach: A case study in Sydney, Australia. *Journal of Cleaner Production*, 280:Article ID 124138, 2021.
[31] V. F. Yu, G. Aloina, H. Susanto, M. K. Effendi, and S. W. Lin. Regional location routing problem for waste collection using hybrid genetic algorithm-simulated annealing. *Mathematics*, 10(12):Article ID 2131, 2022.
[32] J. Zhou, M. Zhang, and S. Wu. Multi-objective vehicle routing problem for waste classification and collection with sustainable concerns: the case of Shanghai city. *Sustainability*, 14(18):Article ID 11498, 2022.